

Rare Disasters and the Term Structure of Interest Rates*

Jerry Tsai

University of Oxford

Abstract

This paper offers an explanation for the properties of the nominal term structure of interest rates and time-varying bond risk premia based on a model with rare consumption disaster risk. In the model, expected inflation follows a mean reverting process but is also subject to possible large (positive) shocks when consumption disasters occur. The possibility of jumps in inflation increases nominal yields and the yield spread, while time-variation in the inflation jump probability drives time-varying bond risk premia. Predictability regressions offer independent evidence for the model's ability to generate realistic implications for both the stock and bond markets.

*First draft: May 21, 2012. This draft: January 13, 2014. Department of Economics, University of Oxford, Manor Road Building, Oxford, OX1 3UQ. jerry.tsai@economics.ox.ac.uk. I am indebted to my advisors Nikolai Roussanov, Ivan Shaliastovich, Jessica Wachter, and Amir Yaron. I also thank Andrew Abel, M. Yasser Boualam, Stephan Dieckmann, Gregory Duffee, Joao Gomes, Philipp Illeditsch, Dana Kiku, Thien T. Nguyen, Christian Opp, Sang Byung Seo, Robert Stambaugh, Mathieu Taschereau-Dumouchel, Luke Taylor, Andrea Vedolin, Colin Ward and seminar participants at Bocconi, Federal Reserve Board, Oxford, Rutgers Business School, Temple, and Wharton for helpful comments. Any errors are my own.

1 Introduction

Empirical work has documented the failure of the expectations hypothesis (Fama and Bliss (1987)). The average nominal term structure of interest rates on government bonds is upward-sloping, and the excess bond returns are predictable. The sizeable and time-varying bond risk premia present challenges to equilibrium economics models, as standard models can not account for these puzzles.

This paper provides a representative agent asset pricing model that can account for these findings. In this model, the endowment is subject to large rare negative shocks (disasters), furthermore, these disasters sometimes co-occurs with large positive jumps on expected inflation.¹ Earlier work has shown that disaster risk models can account for the high equity premium, high stock market volatility and aggregate market return predictability observed in the aggregate stock market.² Besides the aggregate market results shown in previous work, this model can accurately captures nominal bond prices and the time-varying bond risk premia.

The possibility of the co-occurrence of large consumption decline and high inflations has important implication for nominal bond prices. When these disasters happen, nominal bonds have low real value, thus investor requires compensations for bearing these risks. Since bonds with longer maturities are more sensitive to these risks, the model implies a upward-sloping nominal yield curve. With the assumption of recursive utility, the model can also match the level of the term structure and the size of the bond premium. Furthermore, the time-varying nature of disaster probability implies a time-varying bond risk premium, and this model can reproduce the evidence of bond premium predictability documented in Campbell and Shiller (1991) and Cochrane and Piazzesi (2005).

¹An extreme example of these events is the German hyperinflation: Between 1922 and 1923, real consumption declines by 12.7%, and the inflation rate in the corresponding period was 3450%. A comprehensive description of the historical data will be provided later in the model section.

²For example, Rietz (1988), Longstaff and Piazzesi (2004), and Barro (2006) obtain high equity premium, Gabaix (2012), Gourio (2008), and Wachter (2013) also obtain high volatility and predictability.

In the model, each of the two risk factors, the pure consumption disaster risk and the joint consumption-inflation disaster risk, affect both the equity and bond markets. Because of this modeling strategy, the framework proposed in this paper has several advantages over previous work in the term structure literature. First, it provides a parsimonious model that can be calibrated without using bond market data. More specifically, the calibration are targeted to match aggregate consumption growth, inflations, and equity market moments. I then simulate the model to obtain a large number of small-samples. Similar to Bansal, Kiku, and Yaron (2012) and Beeler and Campbell (2012), I construct confidence intervals from the simulations and compare them to postwar U.S. data. We see that the data moments are not only within the confidence interval, but also very close to the median number implied by the model, suggesting the model can generates quantitatively realistic implications for the nominal bond prices and return dynamics.

Second, this paper introduces a novel framework that can account for the interaction between the stock and nominal bond markets. Duffee (2012) suggests that a good predictor for the bond premium may not be a good predictor for the equity premium, and vice versa. In the model, both types of disaster risks induce positive equity premium. On the other hand, while joint consumption-inflation disaster risk carries a positive bond premium, pure consumption disaster risk carries a small and negative bond premium. Given this prices of risk structure, this model can reproduce these empirical results: price-dividend ratio predicts excess returns on the aggregate market (Campbell and Shiller (1988)) and that it has some predictive power for excess returns on the bond market, on the other hand, term structure variables predict excess returns on the nominal bond market (Cochrane and Piazzesi (2005)), yet they are less effective at predicting excess returns on the aggregate market.

Many previous work also provide explanations for the nominal bond prices.³ Among

³For example, Bekaert, Hodrick, and Marshall (2001), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2012), Bakshi and Chen (1996), Dai and Singleton (2002), Duffee (2002), Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), Bikbov and Chernov (2010), Duffee (2006), and Rudebusch and Wu

them, Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013), and Rudebusch and Swanson (2012) also use recursive preferences. Piazzesi and Schneider (2006) focus on the negative effects of surprise inflation on future consumption growth, Bansal and Shaliastovich (2013) build on the Bansal and Yaron (2004) long-run risk framework with stochastic volatility, and Rudebusch and Swanson (2012) extend the long-run risk framework to a DSGE setup. In order to generate the bond premia, these models require higher risk aversion, typically above 20, in the current paper, the risk aversion is set equal to 3.⁴ Furthermore, unlike this paper, these work focus only on the nominal bond market.

Several other papers also provide general equilibrium explanations for both the stock and bond market prices. However, they do not provide realistic implications for the interaction between the two markets. Gabaix (2012) also considers a model with rare disasters. In his work, different processes drive equity and bond premia and the calibration targets both stock and bond market moments.⁵ In contrast, each risk factor in the present model affects both the stock and bond markets, thus the model can be calibrated using stock market data alone, and it provides a realistic framework to examine the interaction between markets. Wachter (2006) and Bekaert, Engstrom, and Grenadier (2010) consider extensions to the model with external habit formation (Campbell and Cochrane (1999)).⁶ These habit formation models imply that the predictable components in the equity and bond premia are high correlated, which is at odds with the data, as Duffee (2012) suggests. Furthermore, habit models implies high relative risk aversion, for example, in Wachter (2006), the risk aversion is above 30 at its long-run mean.

(2008).

⁴Similar to Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013), in this model, when the risk of the co-occurrence of a consumption disaster and high inflations is high, expected consumption growth is low and expectation of expected inflation is high. However, high inflations and low consumption growth only co-occur when this type of consumption disasters are realized.

⁵In that model time-variation in the degree to which dividends respond to a disaster drives the equity premium and time-variation in the size of an inflation jump drives the bond premium.

⁶For habit formation models, see also Buraschi and Jiltsov (2007) and Rudebusch and Swanson (2008), which focus mainly on the term structure of interest rates.

2 Model

2.1 Endowment, inflation, and preferences

The economy is populated with a representative agent. Assume that aggregate real consumption solves the following stochastic differential equation:

$$\frac{dC_t}{C_{t-}} = \mu dt + \sigma dB_{Ct} + (e^{Z_{ct}} - 1) dN_{ct} + (e^{Z_{cq,t}} - 1) dN_{cq,t},$$

where B_{Ct} is a standard Brownian motion. Aggregate consumption is subject to two types of large shocks, and the arrival times of these shocks have a Poisson distribution, given by N_{ct} and $N_{cq,t}$. I will discuss the size and intensity of these Poisson jumps after I specify the inflation process.

To model nominal assets, I assume an exogenous process for the price level:

$$\frac{dP_t}{P_{t-}} = q_t dt + \sigma_P dB_{Pt}, \quad (1)$$

where B_{Pt} is a standard Brownian motion, that is independent of B_{Ct} .

The expected inflation process, q_t , is time-varying. Specifically, it follows

$$dq_t = \kappa_q (\bar{q} - q_t) dt + \sigma_q dB_{qt} - Z_{cq,t} dN_{cq,t} - Z_{qt} dN_{qt}, \quad (2)$$

where B_{qt} is a standard Brownian motion, that is independent of B_{Ct} and B_{Pt} . The expected inflation process is also subject to two types of large shocks, and the arrival time of these shocks follow Poisson distributions, given by $N_{cq,t}$ and N_{qt} . Given these assumptions, in normal times, realized consumption growth and realized inflation are uncorrelated, however, expected consumption growth and expected inflation are negatively correlated as they are both subject to $N_{cq,t}$ -type jumps.

The magnitude of an N_c -type jump is determined by Z_c , the magnitude of an N_{cq} -type jump is determined by Z_{cq} , and that of an N_q -type jump is determined by Z_q . I will

consider all three types of Poisson shocks to be negative, that is $Z_c < 0$, $Z_{cq} < 0$, and $Z_q < 0$; furthermore, these jump sizes are random and have time-invariant distributions ν_c , ν_{cq} , and ν_q , respectively. In what follows, I use the notation E_{ν_j} to denote expectations taken over the distribution ν_j for $j \in \{c, cq, q\}$. The intensities of these Poisson shocks are time-varying, and each follows a square-root process as in Cox, Ingersoll, and Ross (1985). In what follows, I will assume that inflation spike probability is perfectly correlated with inflation disaster probability.⁷ Specifically, for $j \in \{c, cq\}$, the intensity for N_j is denoted by λ_{jt} , and it is given by

$$d\lambda_{jt} = \kappa_{\lambda_j}(\bar{\lambda}_j - \lambda_{jt}) dt + \sigma_{\lambda_j} \sqrt{\lambda_{jt}} dB_{\lambda_{jt}}.$$

$B_{\lambda_{ct}}$ and $B_{\lambda_{cq,t}}$ are independent Brownian motions, and each is independent of B_{Ct} , B_{Pt} , and B_{qt} . Furthermore, assume that the Poisson shocks are independent of each other, and of the Brownian motions. Define $\lambda_t = [\lambda_{ct}, \lambda_{cq,t}]^\top$, $\bar{\lambda} = [\bar{\lambda}_c, \bar{\lambda}_{cq}]^\top$, $\kappa_\lambda = [\kappa_{\lambda_c}, \kappa_{\lambda_{cq}}]^\top$, $B_{\lambda t} = [B_{\lambda_{ct}}, B_{\lambda_{cq,t}}]^\top$, and $B_t = [B_{Ct}, B_{Pt}, B_{qt}, B_{\lambda t}^\top]^\top$.

In what follows, a *disaster* (or *consumption disaster*) is a Poisson shock that affects realized consumption growth. In particular, I will refer to the N_c -type shock as a *non-inflation disaster* and the N_{cq} -type shock as an *inflation disaster*. The N_q -type shock only affects expected inflation and I refer to it as an *inflation spike*. Furthermore, I will refer to λ_c as the *non-inflation disaster probability* and λ_{cq} as the *inflation disaster probability*. Though the latter also governs the intensity of inflation spikes, the majority of its effects comes from inflation disasters rather than inflation spikes.

⁷This assumption means risk of inflation spikes and risk of inflation disaster are perfectly correlated, but they do not necessarily co-occur. Inflation spikes in this model attempt to speak to the period of high inflation in the 1970s and early 1980s. During this period, consumption growth was low, and the outlook for future consumption growth was uncertain. Therefore not modeling inflation spike probability as an independent process is realistic. Furthermore, the discussion in the next section shows that inflation spikes have limited pricing effect, and the main results in this paper does not depend on this process. To keep the model parsimonious, I assume that the inflation spike probability equals inflation disaster probability.

The model setup of is motivated by historical data in Barro and Ursua (2008). Table 1 provides evidence for the co-occurrence of consumption disaster and high inflation. In recorded history, 30 of the 89 consumption disasters were accompanied by inflation rates greater than 10%. Furthermore, these events also happened in developed countries. In fact, 17 of the 53 OECD disasters also co-occurred with a period of high inflation. For example, between 1917 and 1921, real consumption declined by 16.4% in the U.S., and the annual inflation rate during the same period was 13.9%.⁸ Figure 1 shows that the historical distribution of annual inflation rates has a fat tail. Furthermore, these jumps in inflation rates do not happen all at once, they were gradual processes that lasted a number of years.

Following Duffie and Epstein (1992), I define the utility function V_t for the representative agent using the following recursion:

$$V_t = E_t \int_t^\infty f(C_s, V_s) ds, \quad (3)$$

where

$$f(C_t, V_t) = \beta(1 - \gamma)V_t \left(\log C_t - \frac{1}{1 - \gamma} \log((1 - \gamma)V_t) \right). \quad (4)$$

The above utility function is the continuous-time analogue of the recursive utility defined by Epstein and Zin (1989) and Weil (1990), which allows for preferences over the timing of the resolution of uncertainty. Furthermore, equation (4) is a special case when the elasticity of intertemporal substitution (EIS) equals one. In what follows, γ is interpreted as risk aversion and β as the rate of time preference. I assume $\gamma > 0$ and $\beta > 0$ throughout the rest of the paper.

⁸One might argue that consumption disasters are accompanied by large deflation. However, only 17 of the disasters (10 of the OECD disasters) coincide with *any* deflation. Furthermore, none of these disasters had an abnormally large annual deflation rate; for example, the Great Depression had an annual deflation rate of 6.4%. In fact, in an alternative model with empirically plausible deflation disasters, the results reported in this paper remain substantially unchanged.

2.2 The value function and risk-free rates

Let $J(W_t, \lambda_t)$ denote the value function, where W_t denotes the real wealth of the representative agent. In equilibrium $J(W_t, \lambda_t) = V_t$.

Theorem 1. *Assume*

$$(\kappa_{\lambda_c} + \beta)^2 > 2\sigma_{\lambda_c}^2 E_{\nu_{cq}} [e^{(1-\gamma)Z_c} - 1] \quad \text{and} \quad (\kappa_{\lambda_{cq}} + \beta)^2 > 2\sigma_{\lambda_{cq}}^2 E_{\nu_{cq}} [e^{(1-\gamma)Z_{cq}} - 1]. \quad (5)$$

The value function J takes the following form:

$$J(W_t, \lambda_t) = \frac{W_t^{1-\gamma}}{1-\gamma} I(\lambda_t), \quad (6)$$

where

$$I(\lambda_t) = \exp \{a + b_c \lambda_c + b_{cq} \lambda_{cq}\}. \quad (7)$$

The coefficients a and b_j for $j \in \{c, cq\}$ take the following form:

$$a = \frac{1-\gamma}{\beta} \left(\mu - \frac{1}{2} \gamma \sigma^2 \right) + (1-\gamma) \log \beta + \frac{1}{\beta} b^\top (\kappa_\lambda * \bar{\lambda}), \quad (8)$$

$$b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} - \sqrt{\left(\frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} \right)^2 - 2 \frac{E_{\nu_j} [e^{(1-\gamma)Z_j} - 1]}{\sigma_{\lambda_j}^2}}, \quad (9)$$

Here and in what follows, we use $*$ to denote element-by-element multiplication of vectors of equal dimension. The signs of b_c and b_{cq} determine how disaster probabilities λ_c and λ_{cq} affect the investor's value function. The following corollary shows that the investor is made worse by an increase in the disaster probabilities.

Corollary 2. *For $j \in \{c, cq\}$, if $Z_j < 0$, then $b_j > 0$.*

The following two corollaries provide expressions for the real and nominal risk-free rates in this economy.

Corollary 3. Let r_t denote the instantaneous real risk-free rate in this economy, r_t is given by

$$r_t = \beta + \mu - \gamma\sigma^2 + \underbrace{\lambda_{ct} E_{\nu_c} [e^{-\gamma Z_c} (e^{Z_c} - 1)]}_{\text{non-inflation disaster risk}} + \underbrace{\lambda_{cq,t} E_{\nu_{cq}} [e^{-\gamma Z_{cq}} (e^{Z_{cq}} - 1)]}_{\text{inflation disaster risk}}. \quad (10)$$

The terms multiplying λ_{ct} and $\lambda_{cq,t}$ in (10) arise from the risk of a disaster. For $Z_j < 0$, the risk-free rate falls in λ_j : Recall that both non-inflation and inflation disasters affect consumption, therefore high disaster risk increases individuals' incentive to save, and thus lowers the risk-free rate.

Corollary 4. Let $r_t^\$$ denote the instantaneous nominal risk-free rate on the nominal bond in the economy, $r_t^\$$ is given by

$$r_t^\$ = r_t + q_t - \sigma_P^2. \quad (11)$$

The nominal risk-free rate is affected by expected inflation; when expected inflation is high, investors require additional compensation to hold the nominal risk-free asset.

2.3 Nominal government bonds

This section provides expressions for the prices, yields, and premia for nominal zero-coupon government bonds.

2.3.1 Prices and yields

Nominal bond prices are determined using no-arbitrage conditions and the state-price density. Duffie and Skiadas (1994) show that the real state-price density, π_t , equals

$$\pi_t = \exp \left\{ \int_0^t f_V (C_s, V_s) ds \right\} f_C (C_t, V_t), \quad (12)$$

and nominal state-price density, $\pi_t^\$$, is given by⁹

$$\pi_t^\$ = \frac{\pi_t}{P_t}. \quad (13)$$

Let $L_t^{\$, (\tau)} = L^\(q_t, λ_t, τ) denote the time t nominal price of a nominal government bond that pays off one nominal unit at time $t + \tau$. Then

$$L^\$(q_t, \lambda_t, s - t) = E_t \left[\frac{\pi_s^\$}{\pi_t^\$} \right].$$

The price $L_t^{\$, (\tau)}$ can be solved up to four ordinary differential equations. The following corollary is a special case of Theorem B.4 in Appendix B¹⁰

Corollary 5. *The function $L^\$$ takes the following form:*

$$L^\$(q_t, \lambda_t, \tau) = \exp \{ a_L^\$(\tau) + b_{Lq}^\$(\tau)q_t + b_{L\lambda}^\$(\tau)^\top \lambda_t \}, \quad (14)$$

where $b_{L\lambda}^\$(\tau) = [b_{L\lambda_c}^\$(\tau), b_{L\lambda_{cq}}^\$(\tau)]^\top$. The function $b_{Lq}^\$$ takes the form

$$b_{Lq}^\$(\tau) = -\frac{1}{\kappa_q} (1 - e^{-\kappa_q \tau}), \quad (15)$$

the function $b_{L\lambda_c}^\$$ solves

$$\frac{db_{L\lambda_c}^\$}{d\tau} = \frac{1}{2} \sigma_{\lambda_c} b_{L\lambda_c}^\$(\tau)^2 + (b_c \sigma_{\lambda_c}^2 - \kappa_{\lambda_c}) b_{L\lambda_c}^\$(\tau) + E_{\nu_c} [e^{-\gamma Z_{ct}} (1 - e^{Z_{ct}})], \quad (16)$$

⁹Consider a nominal asset that has nominal payoff $X_s^\$$ at time $s > t$, the time t nominal price of the asset, $X_t^\$$, can be written as $X_t^\$ = E_t[\frac{\pi_t}{\pi_s} \frac{P_s}{P_t} X_s^\$] = E_t[\frac{\pi_s^\$}{\pi_t^\$} X_s^\$]$. Therefore, $\pi_t^\$ = \frac{\pi_t}{P_t}$.

¹⁰When the cash flow equals 1, that is, $\mu_{D^s} = 0$, $\sigma_{D^s} = 0$, and $\phi_c^\$ = \phi_{cq}^\$ = 0$.

the function $b_{L\lambda_{cq}}^{\$}$ solves

$$\begin{aligned} \frac{db_{L\lambda_{cq}}^{\$}}{d\tau} = & \frac{1}{2}\sigma_{\lambda_{cq}} b_{L\lambda_{cq}}^{\$}(\tau)^2 + (b_{cq}\sigma_{\lambda_{cq}} - \kappa_{\lambda_{cq}}) b_{L\lambda_{cq}}^{\$}(\tau) \\ & + E_{\nu_{cq}} \left[e^{-(\gamma+b_{Lq}^{\$}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}} \right] + E_{\nu_q} \left[e^{-b_{Lq}^{\$}(\tau)Z_{qt}} - 1 \right], \end{aligned} \quad (17)$$

and the function $a_L^{\$}$ solves

$$\frac{da_L^{\$}}{d\tau} = -\beta - \mu + \gamma\sigma^2 + \sigma_P^2 + \frac{1}{2}\sigma_q^2 b_{Lq}^{\$}(\tau)^2 + b_{Lq}^{\$}(\tau)\kappa_q\bar{q} + b_{L\lambda}^{\$}(\tau)^\top(\kappa_\lambda * \bar{\lambda}), \quad (18)$$

with boundary conditions $a_L^{\$}(0) = b_{Lq}^{\$}(0) = b_{L\lambda_c}^{\$}(0) = b_{L\lambda_{cq}}^{\$}(0) = 0$.

Corollary 5 shows how prices respond to innovations in expected inflation and in changing disaster probabilities. Equation (15) shows that innovations to expected inflation lower prices for nominal bonds of all maturities. Furthermore, the effect will be larger the more persistent it is, that is, the lower is κ_q .

Higher non-inflation disaster probability has a non-negative effect on prices. Consider the ordinary differential equation (16); without the last term $E_{\nu_c} [e^{-\gamma Z_{ct}}(1 - e^{Z_{ct}})]$, the function $b_{L\lambda_c}^{\$}$ is identically zero. Therefore, this term determines the sign of $b_{L\lambda_c}^{\$}$. This term can be rewritten as: $E_{\nu_c} [e^{-\gamma Z_{ct}}(1 - e^{Z_{ct}})] = -E_{\nu_c} [e^{-\gamma Z_{ct}}(e^{Z_{ct}} - 1)]$, which multiplies λ_{ct} in the equation for the nominal risk-free rate (11). Because higher discount rates lower the price, the risk-free rate effect enters with a negative sign. With the boundary condition $b_{L\lambda_c}^{\$}(0) = 0$, this implies that $b_{L\lambda_c}^{\$}(\tau)$ is strictly positive and increasing for all τ . The intuition is straightforward: Non-inflation disaster risks only affect the nominal bonds through the underlying real bonds, and since the real bonds in this economy pay off during consumption disaster periods, they have negative premia.

Unlike non-inflation disasters, the effect of changing inflation disaster probability on bond valuation is more complicated. Recall that this process governs both the probability of an inflation disaster and the probability of an inflation spike. Similarly to the previous argument, the last two terms in ODE (17) determine the sign of $b_{L\lambda_{cq}}^{\$}$. The first expectation

arises from inflation disasters, and it can be rewritten as:

$$\begin{aligned}
& E_{\nu_{cq}} \left[e^{-(\gamma + b_{Lq}^{\$}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}} \right] = \\
& \underbrace{-E_{\nu_{cq}} \left[e^{-\gamma Z_{cq,t}} (e^{Z_{cq,t}} - 1) \right]}_{\text{Risk-free rate effect (-)}} - \underbrace{E_{\nu_{cq}} \left[(e^{-\gamma Z_{cq,t}} - 1)(1 - e^{-b_{Lq}^{\$}(\tau)Z_{cq,t}}) \right]}_{\text{Risk premium effect (+)}} + \underbrace{E_{\nu_{cq}} \left[e^{-b_{Lq}^{\$}(\tau)Z_{cq,t}} - 1 \right]}_{\text{Nominal price effect (-)}}.
\end{aligned} \tag{19}$$

The first component is the risk-free rate effect; as previously discussed, this term is multiplied by a negative sign. The second component is part of the bond premium: The nominal bond price drops during periods of inflation disaster, when marginal utility is high; this term captures the premium investors require for bearing these jump risks. This risk premium effect is also multiplied by a negative sign since an increase in the discount rate lowers the bond price. The last term is the nominal price effect, which represents the effect of change in λ_{cq} on expected nominal bond prices through inflation. More specifically, it is the percent change in the price of a nominal bond with maturity τ in the event of an inflation disaster. Because a higher expected bond value raises the price, this term is multiplied by a positive sign.

Given $\gamma > 0$ and $Z_{cq} < 0$, the risk-free rate effect is negative, the risk premium effect is positive and increasing in maturity τ for $\tau > 0$, and the nominal price effect is negative and decreasing in maturity τ for $\tau > 0$. The effect of changing inflation disaster probabilities on bond value depends on the sum of these three effects. Notice that when $\tau = 0$, only the risk-rate effect is non-zero. Together with the boundary condition $b_{L\lambda_q}^{\$}(0) = 0$, this implies that $b_{L\lambda_q}^{\$}(\tau) > 0$ for some small τ : An increase in inflation disaster probability raises prices on bonds with short maturity. As maturity increases, however, risk premium and nominal price effect prevail over the risk-free rate effect, implying that prices on bonds with longer maturity decrease with inflation disaster probability.

The last term in ODE (17) arises from inflation spike risks. Notice that this term represents the nominal price effect, and it enters with a positive sign. Furthermore, it is

negative and decreasing in maturity τ for $\tau > 0$; implying that an increase in the chance of an inflation spike lowers nominal bond prices and the effect is stronger for bonds with longer maturity.

Before moving on to discuss bond premia, the following definition and corollary provides expression for the nominal bond yield in the model:

Definition 1. *The yield to maturity for a nominal bond with maturity τ at time t , denoted by $y_t^{\$, (\tau)}$, is defined as:*

$$y_t^{\$, (\tau)} = \frac{1}{\tau} \log \left(\frac{1}{L_t^{\$, (\tau)}} \right). \quad (20)$$

Corollary 5 implies that the yield to maturity in this economy takes a particularly simple form:

Corollary 6. *The nominal yield to maturity for a nominal bond with maturity τ at time t , $y_t^{\$, (\tau)}$, is given by*

$$y_t^{\$, (\tau)} = -\frac{1}{\tau} \left(a_L^{\$}(\tau) + b_{Lq}^{\$}(\tau)q_t + b_{L\lambda}^{\$}(\tau)^\top \lambda_t \right), \quad (21)$$

where the coefficients $a_L^{\$}(\tau)$, $b_{Lq}^{\$}(\tau)$, and $b_{L\lambda}^{\$}(\tau)$ are given by (15) - (18).

2.3.2 The bond premium

This section provides an expression for the instantaneous bond premium and discusses its properties. For notation simplicity, I will first define the *jump operator*, which denotes how a process responds to the occurrence of a jump. Let X be a jump-diffusion process. Define the jump operator of X with respect to the j th type of jump as the following:

$$\mathcal{J}_j(X) = X_{t_j} - X_{t_{j-}} \quad j \in \{c, cq, q\},$$

for t_{j-} such that a type- j jump occurs. Then define

$$\bar{\mathcal{J}}_j(X) = E_{\nu_j} [X_{t_j} - X_{t_{j-}}] \quad j \in \{c, cq, q\}.$$

The instantaneous nominal expected return on a nominal bond with maturity τ is simply the expected percent change in nominal prices. Let $L_t^{\$, (\tau)} = L^{\$}(q_t, \lambda_t, \tau)$ be the time- t price of a τ -year nominal bond, by Ito's Lemma:

$$\frac{dL_t^{\$, (\tau)}}{L_t^{\$, (\tau)}} = \mu_{L^{\$, (\tau)}, t} dt + \sigma_{L^{\$, (\tau)}, t} dB_t + \sum_{j \in \{c, cq, q\}} \frac{\mathcal{J}_j(L_t^{\$, (\tau)})}{L_t^{\$, (\tau)}} dN_{jt}.$$

Then the instantaneous expected return can be written as:

$$r_t^{\$, (\tau)} = \mu_{L^{\$, (\tau)}, t} + \frac{1}{L_t^{\$, (\tau)}} \left(\lambda_{ct} \bar{\mathcal{J}}_c(L_t^{\$, (\tau)}) + \lambda_{cq, t} \left(\bar{\mathcal{J}}_{cq}(L_t^{\$, (\tau)}) + \bar{\mathcal{J}}_q(L_t^{\$, (\tau)}) \right) \right). \quad (22)$$

Corollary 7. *The bond premium relative to the risk-free rate $r^{\$}$ is:*

$$r_t^{\$, (\tau)} - r_t^{\$} = - \sum_{j \in \{c, cq\}} \lambda_{jt} b_{L\lambda_j}^{\$}(\tau) b_j \sigma_{\lambda_j}^2 + \lambda_{cq, t} E_{\nu_{cq}} \left[(e^{-\gamma Z_{cq, t}} - 1) (1 - e^{-b_{L\lambda_q}^{\$}(\tau) Z_{cq, t}}) \right] \quad (23)$$

The first term in (23) arises from time-varying non-inflation and inflation disaster probabilities (time-varying probability adjustment). Recall that $b_j > 0$ for $j \in \{c, cq\}$, $b_{L\lambda_c}^{\$}(\tau) > 0$ for all τ , $b_{L\lambda_{cq}}^{\$}(\tau) > 0$ for small τ and $b_{L\lambda_{cq}}^{\$}(\tau) < 0$ for larger τ . Therefore, the time-varying non-inflation disaster probability adjustment is negative because the underlying real bond provides a hedge against consumption disasters. On the other hand, the time-varying inflation disaster probability adjustment is negative for bonds with shorter maturities and positive for bonds with longer maturities. The second term arises from the co-movement in nominal bond prices and marginal utility when a disaster occurs. Notice that this term depends on $b_{Lq}^{\$}$: When an inflation disaster occurs, expected inflation rises, which pushes future bond prices down. Given that $b_{Lq}^{\$} < 0$ and the assumption that $\gamma > 0$, $Z_{qt} < 0$, the second term is positive.

In a sample without disasters, but possibly with inflation spikes, the observed return is

$$r_{nd, t}^{\$, (\tau)} = \mu_{L^{\$, (\tau)}, t} + \frac{1}{L_t^{\$, (\tau)}} \lambda_{cq, t} \bar{\mathcal{J}}_{cq}(L_t^{\$, (\tau)}),$$

where the subscript “nd” is used to denote expected returns in a sample without consumption disasters. The following corollary calculates these expected returns.

Corollary 8. *The expected bond excess returns observed in a sample without disaster is:*

$$r_{\text{nd},t}^{\$(\tau)} - r_t^{\$} = - \sum_{j \in \{c, cq\}} \lambda_{jt} b_{L\lambda_j}^{\$(\tau)} b_j \sigma_{\lambda_j}^2 + \lambda_{cq,t} E_{\nu_{cq}} \left[e^{-\gamma Z_{cq,t}} (1 - e^{-b_{Lq}^{\$(\tau)} Z_{cq,t}}) \right]. \quad (24)$$

2.4 The aggregate market

Let D_t denote the dividend on the aggregate market. Assume that total dividends in the economy evolve according to

$$\frac{dD_t}{D_{t-}} = \mu_D dt + \phi \sigma dB_{Ct} + (e^{\phi Z_{ct}} - 1) dN_{ct} + (e^{\phi Z_{qt}} - 1) dN_{cq,t}. \quad (25)$$

Under this process, aggregate dividend responds to disasters by a greater amount than aggregate consumption does (Longstaff and Piazzesi (2004)). The single parameter, ϕ , determines how aggregate dividend responds to both normal and disaster shocks. In what follows, ϕ is referred to as leverage as it is analogous to leverage in Abel (1999).

Let $H(D_t, \lambda_t, \tau)$ denote the time t price of a single future dividend payment at time $t + \tau$. Then

$$H(D_t, \lambda_t, s - t) = E_t \left[\frac{\pi_s}{\pi_t} D_s \right],$$

where π is the real state-price density defined by (12). The price H can be solved in closed-form up to three ordinary differential equations, and the following corollary is a special case of Theorem B.2 in Appendix B.¹¹

Corollary 9. *The function H takes the following form:*

$$H(D_t, \lambda_t, \tau) = D_t \exp \left\{ a_\phi(\tau) + \lambda_t^\top b_{\phi\lambda}(\tau) \right\}, \quad (26)$$

¹¹ $\phi_c = \phi_{cq} = \phi$, and $\sigma_D = \phi\sigma$.

where $b_{\phi\lambda} = [b_{\phi\lambda_c} b_{\phi\lambda_{cq}}]^\top$. For $j \in \{c, cq\}$, function $b_{\phi j}$ takes the following form:

$$b_{\phi j}(\tau) = \frac{2E_{\nu_j} [e^{(1-\gamma)Z_{jt}} - e^{(\phi-\gamma)Z_{jt}}] (1 - e^{-\zeta_{b_j}\tau})}{(\zeta_{b_j} + b_j\sigma_j^2 - \kappa_j) (1 - e^{-\zeta_{b_j}\tau}) - 2\zeta_{b_j}}, \quad (27)$$

where

$$\zeta_{b_j} = \sqrt{(b_j\sigma_j^2 - \kappa_j)^2 + 2\sigma_j^2 E_{\nu_j} [e^{(1-\gamma)Z_{jt}} - e^{(\phi-\gamma)Z_{jt}}]}. \quad (28)$$

Function $a_\phi(\tau)$ takes the following form:

$$a_\phi(\tau) = \left(\mu_D - \mu - \beta + \gamma\sigma^2(1 - \phi) - \left(\sum_{j \in \{c, cq\}} \frac{\kappa_{\lambda_j} \bar{\lambda}_j}{\sigma_{\lambda_j}^2} (\zeta_{b_j} + b_j\sigma_{\lambda_c}^2 - \kappa_{\lambda_j}) \right) \right) \tau - \sum_{j \in \{c, cq\}} \left(\frac{2\kappa_{\lambda_j} \bar{\lambda}_j}{\sigma_{\lambda_j}^2} \log \left(\frac{(\zeta_{b_j} + b_j\sigma_{\lambda_j}^2 - \kappa_{\lambda_j})(e^{-\zeta_{b_j}\tau} - 1)}{2\zeta_{b_j}} \right) \right). \quad (29)$$

Let $F(D_t, \lambda_t)$ denote the time t price of the aggregate stock market (the price of the claim to the entire future dividend stream). Then

$$F(D_t, \lambda_t) = \int_0^\infty H(D_t, \lambda_t, \tau) d\tau.$$

Equation (27) shows that $b_{\phi j}(\tau) < 0$ for $j \in \{c, cq\}$; therefore the price-dividend ratio,

$$G(\lambda_t) = \int_0^\infty \exp \{ a_\phi(\tau) + \lambda_{ct} b_{\phi\lambda_c}(\tau) + \lambda_{cq,t} b_{\phi\lambda_{cq}}(\tau) \} d\tau, \quad (30)$$

decreases in both non-inflation and inflation disaster probability.

Using Ito's Lemma, the process for the aggregate stock price $F_t = F(D_t, \lambda_t)$ can be written as:

$$\frac{dF_t}{F_t} = \mu_{F,t} dt + \sigma_{F,t} dB_t + \sum_{j \in \{c, cq\}} \frac{J_j(F_t)}{F_t} dN_{jt}.$$

The instantaneous expected return is the expected change in price, plus the dividend yield:

$$r_t^m = \mu_{F,t} + \frac{D_t}{F_t} + \sum_{j \in \{c,cq\}} \lambda_{jt} \frac{\bar{J}_j(F_t)}{F_t}. \quad (31)$$

Corollary 10. *The equity premium relative to the risk-free rate is:*

$$r_t^m - r_t = \phi\gamma\sigma^2 - \sum_{j \in \{c,cq\}} \lambda_{jt} \frac{1}{G_t} \frac{\partial G_t}{\partial \lambda_j} b_j \sigma_{\lambda_j} + \sum_{j \in \{c,cq\}} \lambda_{jt} E_{\nu_j} [(e^{-\gamma Z_j} - 1)(1 - e^{\phi Z_j})]. \quad (32)$$

Proof of this Corollary is similar to the proof of Corollary 7.

3 Quantitative results

The model is calibrated to match aggregate consumption growth, inflation, and aggregate market moments. To evaluate the quantitative implication of the model, I discretize the model at monthly frequency and simulate monthly data for 60,000 years. Furthermore, I simulate 100,000 60-year samples, also at monthly frequency. For each of these small-samples, the initial values of λ_{ct} and $\lambda_{cq,t}$ are drawn from their stationary distributions, and the initial value of q_t is set equal to its mean, \bar{q} . In each of the tables that follow, I report the data and population value for each statistic. In addition, I report the 5th-, 50th-, and 95th-percentile values from the small-sample simulations (labelled “All Simulations” in the tables), and the 5th-, 50th-, and 95th-percentile values for the subset of the small-sample simulations that do not contain disasters (labelled “No-Disaster Simulations” in the tables). Samples in this subset do not contain any jumps in consumption, but they may contain jumps in expected inflation.

In the past 60 years, the U.S. did not experience any consumption disasters; however, it experienced a period of high inflation in the late 1970s and early 1980s. The No-Disaster subset from the simulation accommodates the possibility that there was an inflation jump in the country’s postwar history; statistics from this subset therefore offer the most interesting

comparison for the U.S. postwar data. With this calibration, about 23% of the samples do not experience any type of consumption disaster, and about one-third of these samples contain at least one jump in expected inflation.¹²

3.1 Calibration

3.1.1 Data

The data on bond yields are from the Center for Research in Security Prices (CRSP). Monthly data is available for the period between June 1952 and December 2011. The yield on the three-month government bills is from the Fama risk-free rate, and yields on zero-coupon bonds with maturities between one and five year are from the Fama-Bliss discount bond dataset.

The market return is defined as the gross return on the CRSP value-weighted index. The dividend growth rate is from the dividends on the same index. To obtain real return and dividend growth, I adjust for inflation using changes in the consumer price index, which is also available from CRSP. The price-dividend ratio is constructed as the price divided by the previous 12 months of dividends. The government bill rate is the inflation-adjusted three-month Treasury Bill return. All data are annual. I use data from 1947 to 2010; using only postwar data provides a comparison between U.S. data and the simulated samples without consumption jumps.

3.1.2 Parameter values

Table 2 reports the parameter values. Mean consumption growth is set at 1.96% and the volatility of consumption growth is 1.45% to match their postwar data counterparts. Mean dividend growth is set to 3.63% to match the price-dividend ratio. The leverage parameter ϕ governs the ratio between the volatility of log dividends and the volatility

¹²As previously mentioned, inflation spikes have limited pricing effect. In an alternative model without inflation spikes, the main results remain substantially unchanged.

of log consumption, which suggest the value to be 4.66. However, it also determines how dividends response to consumption disasters and hence the equity premium. I choose, $\phi = 3.5$, so that dividends have a more conservative response to consumption disasters and generates a sizeable equity premium. Rate of time preference β is set to be low to obtain a realistic short-term government bill rate. Relative risk aversion γ is set equal to 3.

Mean expected inflation is set to 2.7%; with this value, the median value of the realized inflation among the simulations with no consumption disaster is 3.72%, and the data counterpart is 3.74%. The volatility of non-expected inflation σ_p equals 0.8% to match the realized inflation volatility in the data; the median value among the simulations with no consumption disaster is 2.94%, and the value in the data is 3.03%. The volatility of expected inflation σ_q equals 1.3% to match the volatility of short-term bond yield; the volatility of three-month Treasury Bill yield is 3.01% in the data, and the median value among the simulations with no consumption disaster is 3.03%. The mean reversion parameter in the expected inflation process governs the persistence of the inflation process, which is highly persistent and the autocorrelation decays slowly. This parameter it is set to 0.09 to obtain a reasonable first order autocorrelation of the inflation process.

Barro and Ursua (2008) calibrated the average probability of a consumption disaster for OECD countries to be 2.86%, implying that $\bar{\lambda}_c + \bar{\lambda}_{cq} = 2.86\%$.¹³ In the data, about one-third of the disasters are accompanied by high inflation (Table 1), therefore I set $\bar{\lambda}_c$ to equal 1.83% and $\bar{\lambda}_{cq}$ to equal 1.03%. The persistence in the price-dividend ratio is mostly determined by the persistence in the disaster probability, therefore the mean reversion parameters for both inflation and non-inflation disaster probabilities are set to be low: $\kappa_{\lambda_c} = \kappa_{\lambda_{cq}} = 0.11$. With this choice, the median value of the persistence of the price-dividend ratio among the simulations with no consumption disaster is 0.75; the value in the data is 0.92. The volatility parameters of the intensity processes are chosen to obtain

¹³I calibrate the disaster probability to the OECD subsample but the size of jumps to the full set of samples. This is a more conservative approach as OECD countries have less frequent but more severe disasters.

reasonable volatility for the aggregate market returns. Given other parameters, Equation 9 imposes upper bounds for these values, I set σ_{λ_c} and $\sigma_{\lambda_{cq}}$ are set at the maximum value: $\sigma_{\lambda_c} = 0.107$ and $\sigma_{\lambda_{cq}} = 0.093$. With these values, the median values of aggregate market return volatility in the no disaster samples is 14.59%, the postwar data counterpart is 17.79%.

The disaster distributions Z_c and Z_{cq} are chosen to match the distribution of consumption declines in all historical disaster events. Following Barro (2006) and Barro and Jin (2011), I consider 10% as the smallest possible disaster magnitude and I assume that jump sizes follow power law distributions. For non-inflation disasters, I set the power law parameter to equal 9, and for inflation disasters, I set the power law parameter to equal 7. In the data, the distribution of consumption decline has mean 26% and median 19% for inflation disasters, in the model, the mean and median are 27% and 22%, respectively. For non-inflation disasters, the mean is 20% and the median is 16% in the data, and 22% and 19% in the model. Figure 2 plots these power law distributions along with distributions of large consumption declines. It compare the power law distribution with parameter 7 to the distribution of large consumption declines that are accompanied by high inflation, and the power law distribution with parameter 9 to the distribution of large consumption declines that are not accompanied by high inflation. In addition, I will assume that Z_q follows the same distribution as Z_{cq} .¹⁴

3.2 Yield curves and expected returns as functions of the state variables

Figure 3 plots the loadings functions in the expression for the nominal bond yield (21), using parameters in Table 2. In particular, it shows the loading on expected inflation, $-b_{Lq}/\tau$; on non-inflation disaster probability, $-b_{L\lambda_c}/\tau$; and on the inflation disaster probability,

¹⁴Figure 1 shows that inflation rates during disasters has a fatter tail than consumption declines. Hence this assumption truncates the extreme events.

$-b_{L\lambda_{cq}}/\tau$; all as functions of maturity τ .

The loading on expected inflation is positive and decreases with maturity: High expected inflation lowers bond values and raises bond yields; due to mean-reversion, the effect is slightly larger on bonds with shorter maturities. The loading on non-inflation disaster probability is negative and decreases with maturity. While the loading on the inflation disaster probability is also negative for short maturity bonds, it increases with maturity and becomes positive. As discussed in Section 2.3.1, non-inflation disaster risks only induce risk-free rate effect: High non-inflation disaster risk lowers the risk-free rate, and leads to higher bond prices and lower bond yields. Therefore, the coefficient on the non-inflation disaster probability is negative and decreasing with maturity. On the contrary, because inflations jump up during inflation disaster times, inflation disaster probability also have risk premium effect and nominal price effect, both lead to lower bond prices and higher bond yields.¹⁵ The bottom-right panel of Figure 3 shows that the risk-free rate effect dominates for short maturity bonds while the risk premium and nominal price effect dominates for bonds with longer maturities. Furthermore, the shape of function $-b_{L\lambda_{cq}}/\tau$ is mainly determined by the risk premium effect. From this figure, one can also observe that the slope of the yield curve is mainly determined by inflation disaster probability.

Figures 4 and 5 plots the bond risk premia as functions of non-inflation disaster probability, λ_c , and inflation disaster probability, λ_q , using Equation (23) and parameters in Table 2. Expected inflation q is set equal to 2.8% in all cases. To illustrate the impact of changes in disaster probabilities on bonds with different maturities, I compare the risk premia on one- and five-year bonds. Figure 4 shows that risk premia decrease with non-inflation disaster probability, and also that bonds with longer maturities are more sensitive to these changes. Equation (23) shows that non-inflation disaster probability implies a negative premium, and that the absolute magnitude of this premium increases with maturity. Figure 5 shows that risk premia increase as a function of the inflation disaster probability and that bonds with longer maturities are more sensitive to this risk: The co-movement of

¹⁵Recall that part of the nominal price effect comes from the presence of inflation spike risks.

marginal utility and bond prices in inflation disaster periods generates a positive premium for all nominal bonds, and this premium increases with maturity. While non-inflation and inflation disaster risks has the opposite effect on bond premium, by comparing Figures 4 and 5, one can also see that bond risk premia are more sensitive to inflation disaster risks than to non-inflation disaster risks. This suggests that most of the time-variation in bond risk premia comes from variation in inflation disaster risks.

Figure 3 – 5 provide evidence of predictable bond premia in the model. Figure 3 shows that high inflation disaster risk leads to high yield spread, and Figure 4 and 5 show that high inflation disaster risk also leads to high bond premium.¹⁶ Therefore, one should expect yield spread to have predictive power on bond premium in this model: Higher yield spread are likely to be followed by higher bond excess returns. Furthermore, since premium on long-term bonds are more sensitive to disaster probabilities than premium on short-term bonds, longer-term bond excess returns should be more sensitive to changes in yield spreads than excess returns of shorter-term bonds are.

3.3 Simulation results

3.3.1 Nominal yields and returns

Figures 6 and 7 show the first two moments of yields for nominal bonds with different maturities. Figure 6 plots the data and model-implied average nominal bond yields, and Figure 7 plot the data and model-implied volatility of nominal bond yields, both as functions of time to maturity. In each figure, I plot the median, the 25th-, and 75th-percentile values from the subset of small-samples that do not contain any consumption disasters. Table 5 reports the data and all model-implied statistics of mean and volatility of yields for nominal bonds with one, two, three, four, and five years to maturity.

The model is capable of explaining the level and shape of the average nominal yield curve. The median values from the no-disaster simulations increase with maturity and are

¹⁶Also true for low non-inflation disaster risk.

close to their data counterparts. The median value of the average bond yields increases from 5.53% for one-year bonds to 6.06% for five-year bonds; in the data, it increase from 5.20% for one-year bonds to 5.82% for five-year bonds. In addition to the first moment, the model also generates realistic implications for the volatility of bond yields. The median values decreases from 2.87% for one-year bond to 2.69% for five-year bond; in the data, it decreases from 3.02% for one-year bond to 2.78% for five-year bond. The confidence intervals for these results are large because these samples contains periods with inflation spikes.

The nominal yields are on average higher and more volatile in the full set of simulations and in population because these simulations contain more jumps on expected inflation.¹⁷ Another observation from these table is that the population moments in these tables are much higher than the median of the small sample simulations, and this is because inflations rates for the small samples are highly skewed. Inflation jumps occur less frequently in the median small sample than in population, and about 36% of the small samples do not contains any inflation jumps. In fact, the population moments are close the average of the small sample.¹⁸

Previous consumption-based models successfully capture the level and shape of the nominal yield curve. However, unlike this model, they do not generate realistic implication for the second moment. The habit formation model in Wachter (2006) implies that short-term yields are more volatile than long-term yields, which is counterfactual. In both Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013), short-term bond yields are also more volatile than long-term bond yields, but the levels are much lower than the data counterparts.¹⁹ Comparing with the three models, this model also impose a poten-

¹⁷In population, jumps on expected inflation occur about 2% of the times. The median number of jumps on expected inflation in the full set of small samples is 1, which is about 1.67%, and the median number of inflation jump in the no-disaster samples is 0.

¹⁸The inflation moments in Table 4 can be interpreted the same way.

¹⁹For example, in Bansal and Shaliastovich (2013), the yield volatility decreases from 2.37% for one-year bond to 2.17% for five-year bond.

tially more reasonable requirement on the utility function of the representative agent. In the current calibration, relative risk aversion is set equal to 3. In contrast, in order to generate sizeable premium, Piazzesi and Schneider (2006) set it equal to 43 and Bansal and Shaliastovich (2013) estimate it to be 20.90. The habit formation model in Wachter (2006) assumes a time-varying risk aversion, which is greater than 30 when the state variable is at its long-run mean.

Besides nominal bond yields, this model also generates realistic moments for bond returns. Table 6 reports the mean and standard deviation of annual excess returns on bonds with two, three, four, and five years to maturity, all relative to returns on one-year bonds. Panel A and B of the table shows that excess returns on longer term bonds are on average larger, and more volatile. Average bond excess returns increases from 0.35% for two-year bond to 0.97% for five-year bond, and excess return volatility increase from 1.76% for two-year bond to 6.66% for five-year bond. As mentioned in the earlier section, bond premium in the model is driven mainly by inflation disaster probability, and this risk affect long term bonds more than short term bonds. Together, these implies the Sharpe ratio in the model decreases with bond maturity, from 0.19 for two-year bond to 0.14 for five-year bond. Evidence on the inverse relation between Sharpe ratio and bond maturity is also provided by Campbell and Viceira (2001), and Duffee (2010).

3.3.2 Principal component analysis

Litterman and Scheinkman (1991) find that most of the variations in yield curve can be explained by a three-factor model. Specifically, the first factor affects the level of the yield curve, the second factor affects the slope, and the third factor affects the curvature. To evaluate whether the model also exhibits this feature, I perform a principal component analysis on the data and model-simulated yields. Figure 9 reports the results. For the model, I only report the median values drawn from the subset of small-sample simulations that does not contain any disasters. I plot the loadings on yields with different maturities on each of the first three principal components. Similar to Litterman and Scheinkman

(1991), a shock to the first principal component has similar effects across yields of different maturities (*level factor*); a shock to the second principal component raises yields on short-term bonds and reduces yields on long-term bonds (*slope factor*); and a shock to the third principal component raises yields on bonds with median maturity, but lowers yields on short- and long-term bonds (*curvature factor*). In addition, the bottom-right panel also shows that almost all the variations in yield curve are explained by the first three principal components, both in the data and in the model.

Given the three-factor structure of the model, it is natural to ask how these three factors relate to the three state variables in the model. Table 7 reports the correlation between each of the three state variables in the model and each of the three principal components. The level factor is mostly correlated with expected inflation, with correlation coefficient equals .96; consistent with Figure 3, an increase in expected inflation or inflation disaster probability raises the yield curve, while an increase in non-inflation disaster probability lowers it. The slope factor is highly negatively correlated with the inflation disaster probability, with correlation coefficient $-.95$ and slightly positively correlated with expected inflation and non-inflation disaster probability.²⁰ The curvature factor is mostly correlated with non-inflation disaster risks; a shock to non-inflation disaster risks (also expected inflation and inflation disaster risks) increase the curvature of the yield curve.

3.3.3 Time-varying bond risk premia

First I consider the “long-rate” regression in Campbell and Shiller (1991):

$$y_{t+h}^{\$, (n-h)} - y_t^{\$, (n)} = \text{constant} + \beta_n \frac{1}{n-h} \left(y_t^{\$, (n)} - y_t^{\$, (h)} \right) + \text{error}, \quad (33)$$

where n denotes bond maturity and h denotes the holding period. Under the expectations hypothesis, excess returns on long-term bonds are unpredictable, which implies that β_n

²⁰Note that the loadings on the second principal component decrease with maturity, so a positive shock to this principal component reduces the slope.

should equal one for all n . In what follows, I will consider this regression at quarterly frequency using long-term bonds with maturities of one, two, three, four, and five years, that is, $n = 1, 2, 3, 4, 5$ and $h = 0.25$. Figure 8 and Table 8 reports the results of regression (33). Using data from June 1952 to December 2011, the coefficient β_n 's are negative for all maturities n , and these coefficients decrease from -0.57 for one-year bond to -1.68 for five-year bond. This implies that higher yield spread predicts higher bond excess return, furthermore, a high yield spread predicts a higher excess return for bonds with longer maturity.

The model is also capable of capturing this feature. Equations (23) and (24) show that higher inflation disaster risks and lower non-inflation disaster risks lead to higher bond risk premia, furthermore, Figure 4 and 5 show that the effect of disaster risks on bond premia increase with maturity. On the other hand, Figure 3 shows that higher inflation disaster risks and lower non-inflation disaster risks also lead to higher yield spreads. These imply that bond premia are high when yield spread is large, and given the level of yield spread, bond premia increases with maturity. As shown in Table 8, the median values of the coefficients among the simulations that contain no consumption disasters are also negative and decreasing with maturity n , from -0.30 for one-year bond to -0.66 for five-year bond. Furthermore, the data values are all above the 5th percentile of the values drawn from the model. Higher inflation disaster risks, however, also lead to a higher probability of jumps on expected inflations. Once these jumps are realized, bond prices drop and realized excess returns fall. In Table 8, one can see that while the median values drawn from the no-disaster simulations are negative, the population values and the median values among the full set of simulations are positive.

In addition to the long-rate regressions, I also consider the forward rate regressions performed by Cochrane and Piazzesi (2005) to evaluate the model's success in capturing time-varying bond risk premia. In what follows, I consider the annual forward rate. The

n -year forward rate at time t for a loan from time $t + n$ to time $t + n + 1$ is denoted by f_t^n :

$$f_t^n = \log L_t^{\$, (n-1)} - \log L_t^{\$, (n)}.$$

The forward rate regressions are done in two steps. First I regress the average annual excess returns on two-, three-, four-, and five-year nominal bonds on all available forward rates:

$$\frac{1}{4} \sum_{n=2}^5 r_{t+1}^{e, (n)} = \theta^\top \mathbf{f}_t + \text{error}, \quad (34)$$

where $r_{t+1}^{e, (n)} = r_{t+1}^{\$, (n)} - r_{t+1}^{\$, (1)}$ is the excess return of a bond with maturity n and \mathbf{f}_t denotes the vector of all forward rates available at time t . The second step is to form a single factor $\hat{\mathbf{c}}_{t+1} = \hat{\theta}^\top \mathbf{f}_t$ and regress the excess returns of bonds with different maturities on this single factor:

$$r_{t+1}^{e, (n)} = \text{constant} + \rho_n \hat{\mathbf{c}}_{t+1} + \text{error}. \quad (35)$$

Following Cochrane and Piazzesi, I consider monthly overlapping annual observations from June 1952 to December 2011. In the data, I construct one-, two-, three-, four-, and five-year forward rates. In the model, however, I can only construct three independent forward rates since the model only has three factors. Therefore, I will use all five forward rates in the data, but only one-, three-, and five-year forward rates in the model.

Table 9 reports the results from the second stage regression, (35). In the data the single forward rate factor predicts bond excess returns with an economically significant R^2 , between 0.15 and 0.20. Furthermore, the coefficient on this factor increases with bond maturity, from 0.44 for two-year bonds to 1.47 for five-year bonds, implying that the forward rate factor predicts higher return for longer term bonds. The model successfully generates these findings: The median values of the R^2 's drawn from the subset of the small-sample simulations containing no consumption disasters are around 0.18, which are close to their data counterparts. The median value of the coefficients in these samples also increase with maturity: from 0.42 for two-year bonds to 1.57 for five-year bonds, also match their

data counterparts. In the full set of small-samples, the R^2 are lower but still economically meaningful, the population R^2 however, are almost zero.

In the model, the forward rate factor is a linear combination of the state variables, with the inflation disaster risk being the more important one: the median value of the coefficient in the sample without consumption disaster is 1.54 for λ_{cq} and -0.42 for λ_c . The model is therefore able to replicate the empirical finding because the linear combination of forward rates is mostly determined by the inflation disaster probability, and higher inflation disaster probability leads to a higher forward rate factor and higher bond risk premia, and the premia increases with bond maturity²¹.

3.3.4 The aggregate market

This model is also successful in matching moments in the aggregate market. Table 10 reports the simulation results. The model is able to explain most of the equity premium, which is 7.25% in the data; the median value from the no-disaster subsample is 6.12%, and the data is below the 95th percentile of the values drawn from the model. The volatility of equity returns equals 17.79% in the data, and the median value in the model is 14.59%. This model generates a Sharpe ratio of 0.42, which is also close to its postwar data counterpart, 0.41.

To calculate the real three-month Treasury Bill returns, I calculate the realized returns on the nominal three-month Treasury Bill, then adjust them by realized inflation. This model generates reasonable values for the short-term interest rate; this value in the data is 1.25%, and the median value from the small-sample containing no consumption disasters is 1.80%. Furthermore, the data value is above the 5th percentile of the values drawn from the model, indicating that we cannot reject the model at the 10% level.

The model, however, only has limited ability to explain the volatility of the price-dividend ratio. As discussed in Bansal, Kiku, and Yaron (2012) and Beeler and Campbell

²¹Variation in non-inflation disaster probability also leads to similar results, as high non-inflation disaster probability leads to lower forward rate factor and lower risk premia.

(2012), this is a limitation shared by models that explain aggregate prices using time-varying moments but parsimonious preferences. Time-varying moments imply cash flow, risk-free rate, and risk premium effects, and one of these generally acts as an offset to the other two, thus limiting the effect time-varying moments have on prices.

3.3.5 Interactions between the aggregate and bond market

Previous works have shown that variables that predict excess returns in one asset class often fail in another. For example, Duffee (2012) showed that while term structure variables predict bond excess returns, they do not predict stock market excess returns, and vice versa. In this section, I consider two predictor variables, the price-dividend ratio and the linear combination of forward rates that best predicts bond returns. I also consider two excess returns, the aggregate market excess return, defined as aggregate market returns over short term bill returns, and the average bond returns, defined as the average of the returns on one-, two-, three-, four- and five-year nominal bonds returns over the short term bill returns. I calculate the predictive regressions of each excess returns on each predictor variable, both in the model and using annual data from 1953 to 2010. Tables 11 – 14 report the results from these predictive regressions.

Tables 11 and 13 show the results of regressing aggregate and bond market excess returns on the price-dividend ratio. Panel A of these tables shows the result for aggregate market excess returns. It is well known that price-dividend ratio predicts aggregate market excess returns in the data (e.g. Campbell and Shiller (1988), Cochrane (1992), Fama and French (1989) and Keim and Stambaugh (1986)), and similar to previous work in the time-varying disaster risk literature, this model is able to generate this feature. Equation (30) shows that high disaster probability lowers the price-dividend ratio. Furthermore, when disaster risk is high, investors require a high premium to hold the aggregate market asset. This implies that on average, a low price-dividend ratio is followed by high excess returns. Low price-dividend ratio also implies that disasters are more likely to happen. When disasters occur, dividends and realized returns drop, therefore predictability becomes weaker in the full set

of simulations. Predictability is even weaker in population, reflecting the small-sample bias in predictive regressions.

In the data, the price-dividend ratio also has some predictive power on long-term bond excess returns, though the coefficients are not statistically significant and R^2 values are low. The model generates similar implications. In the model, inflation disaster and non-inflation disaster risks have similar effect on the price-dividend ratio: Higher disaster risks, higher price-dividend ratio. However, the two types of disaster risks have different effect on the bond premium. High inflation disaster leads to a large positive bond premium while high non-inflation disaster risk leads to a small negative bond premium. Because the two risks have offsetting effects, price dividend is not a good predictor for the bond market excess return. The coefficients are close to zero and the R^2 's are low. In the data, price-dividend ratio predicts stock market excess return with $R^2 = 0.13$ and bond market returns with $R^2 = 0.09$ with 5-year holding period, in the model, the R^2 's for stock market return and bond returns in the model are 0.44 and 0.07, respectively.

Tables 12 and 14 report the results of the regressing aggregate and bond market excess returns on the linear combination of forward rates. As shown in the previous section, a linear combination of forward rates can predict long-term bond excess returns, and the model successfully replicate this result. In the model, both the bond premium and the linear combination of forward rates increase with inflation disaster probability and decrease with non-inflation disaster probability. Hence when the linear combination of forward rates is high, the bond premium is also high.

On the other hand, the linear combination of forward rates has less predictive power on the aggregate market excess returns (Duffee (2012)).²² In the model, the two types

²²The magnitude of the R^2 -statistics depends on the subsample. For example, Cochrane and Piazzesi (2005) find that the linear combination of forward rates predicts one-year aggregate market excess returns with an $R^2 = 0.07$ in the sample from 1964 through 2003. In the corresponding period, the R^2 is 0.36 for one-year nominal bond excess returns. In the full sample from 1953 to 2010, the linear combination of forward rates appears to have no predictive power.

of disaster probability affect the aggregate market premium in the same direction, higher disaster probability, higher premium. However, they have different effects on the linear combination of forward rates. Therefore, the linear combination of forward rates is not a good predictor for the aggregate market premium in the model. Comparing Panel A and Panel B of Tables 12 and 14, one can see that the linear combination of forward rates predicts the long-term bond excess returns with a much higher R^2 value than for the aggregate market excess returns, implying that the forward rate factor has a stronger predictive power on bond excess returns. For example, forward rates factor predicts aggregate market excess return with $R^2 = 0.02$ and bond market excess returns with $R^2 = 0.12$ with 5-year holding period in the data. The R^2 's in the model are 0.08 for aggregate market excess return, and 0.32 for bond market excess return.

Lettau and Wachter (2011) also consider these regressions; the single forward rate factor in their model predicts bond excess returns and aggregate market excess returns with similar R^2 values. In the data, even though the R^2 depends on the sample period, the forward rate factor has a stronger predictive power on bond excess returns. Wachter (2006) and Gabaix (2012) also study both the stock and bond markets. The model of Wachter (2006), however, implies that the risk premia on stocks and bonds move together. In Gabaix (2012), the time-varying risks in stock and bond market are unrelated, where in this paper, the underlying risks are the same, but they have different effect on the premia. The model in this paper is able to generate more realistic implications for these predictive regressions because the prices of risks in the model have a two-factor structure, and these factors have differential effects on the stock and bond markets.

3.3.6 Implied time series

The three state variables, expected inflation, non-inflation disaster probability and inflation disaster probability, determine equity and nominal bond prices in the model. Therefore, with historical price data, one can find implied values of q_t , $\lambda_{c,t}$, and $\lambda_{cq,t}$ by inverting the equations for nominal bond yield and price-dividend ratio, (21) and (30). In this section,

I will use information from both the equity and bond market to calculate the implied state variables, since equity prices help identifying the total disaster probability and bond prices identifies expected inflations and separate the two types of disaster risks. Following Wachter (2013), rather than using the price-dividend ratio, I use the monthly series of price-earning ratio on the S&P 500, which can be found on Robert Shiller’s website.²³ In order to compare it to the price-dividend ratio in the model, I demean both series by subtracting the sample mean of the price-earning ratio and add the population mean of the price-dividend ratio calculated from the model. I also use monthly average nominal yields and monthly yield spreads (defined as five-year yield minus one year yield). I then calculate the implied values of q_t , $\lambda_{c,t}$, and $\lambda_{cq,t}$ using equations (21) and (30), restricting the disaster probabilities $\lambda_{c,t}$, and $\lambda_{cq,t}$ to be non-negative.

Figure 10 plots the resulting time series of these states variables from June 1952 to December 2012, at monthly frequency. The average expected inflation during this period is 3.43%, the average non-inflation disaster probability is 2.08% and the average inflation disaster probability is 0.75%. Non-inflation disaster probability was relatively more important in the 1950s and the 1960s, and expected inflation was relatively low during this period. Both type of disaster probabilities start rising during the stagflation period in the 1970s. In particular, non-inflation disaster probability peaked at 8% around late 1970s and inflation disaster peaked at 3% in the 1980s. Inflation disaster probability becomes more important, it even briefly exceeds non-inflation disaster probability around that time. During the same period, expected inflation was also rising rapidly, peaking at 17% early 1980s. Both disaster probabilities were low in the 1990s and early 2000s, and expected inflation during this time was less volatile.²⁴

²³Fama and French (2001) points out that firms have been paying less dividend. However, price-dividend ratio in the model can not capture this characteristic, and calculating implied disaster probability using price-dividend ratio can be misleading. Hence I use the price-earning ratio to alleviate this concern.

²⁴Notice that the implied expected inflation series become negative in early 2000s and also during the recent crisis. During these periods, yield levels were relatively low and yield spreads were relatively high. Because expected inflation is the only variable that can move yield level and yield spread in different

To further examine the plausibility of the implied expected inflation series, I look at its correlation with data on expectations of inflation. The forecast data is from the Survey of Professional Forecast and it is available at quarterly frequency from 1970. The correlation between the data and model-implied expected inflations is around 87%, furthermore, this correlation is higher than the correlation between expected inflations and average yield level in the data, which is 78%. These results suggest that the model implies reasonable expected inflations, also, even though it is mostly determined by the level of nominal bond yields, other term structure information also helps identifying expected inflation.

4 Conclusion

Why is the average term structure upward-sloping? Why are excess returns on nominal bonds predictable? This paper provides an explanation for these questions using a model with time-varying rare disaster risks. Previous research has shown that a model that includes time-varying disaster risks can generate high equity premium and excess returns volatility. Motivated by historical data, disasters in this model affect not only aggregate consumption, but also expected inflation. A jump in expected inflation pushes down the real value of nominal bonds, and investors require compensation for bearing these inflation disaster risks. Furthermore, this premium increases with bond maturity, which leads to an upward-sloping nominal term structure. Time-varying bond risk premia arise naturally from time-varying disaster probabilities, and prices of risk in this model follow a two-factor structure.

The model is calibrated to match the aggregate consumption, inflation, and equity market moments, and the quantitative results show that this model produces realistic means and volatilities of nominal bond yields. The three state variables in the model are highly correlated with the first three principal components, which explain almost all of the variations in the nominal yield curve both in the model and in the data. This model can

direction in the model, it leads to negative implied expected inflations.

also account for the violation of the expectations hypothesis. In particular, I show that the yield spread and a linear combination of forward rates can predict long-term bond excess returns. Furthermore, the model is capable of capturing the joint predictive properties of the aggregate market returns and of the bond returns. Aggregate market variables have higher predictive powers for equity excess returns while the term structure variables have higher predictive powers for bond excess returns.

Appendix

A Model derivation

A.1 Notation

Definition A.1. Let X be a jump-diffusion process. Define the jump operator of X with respect to the j th type of jump as the following:

$$\mathcal{J}_j(X) = X_{t_j} - X_{t_{j-}} \quad j \in \{c, cq, q\},$$

for t_{j-} such that a type- j jump occurs. Then define

$$\bar{\mathcal{J}}_j(X) = E_{\nu_j} [X_{t_j} - X_{t_{j-}}] \quad j \in \{c, cq, q\},$$

and

$$\bar{\mathcal{J}}(X) = [\bar{\mathcal{J}}_c(X), \bar{\mathcal{J}}_{cq}(X), \bar{\mathcal{J}}_q(X)]^\top.$$

A.2 The value function

Proof of Theorem 1 Let S denote the value of a claim to aggregate consumption, and conjecture that the price-dividend ratio for the consumption claim is constant:

$$\frac{S_t}{C_t} = l,$$

for some constant l . This relation implies that S_t satisfies

$$dS_t = \mu S_t dt + \sigma S_t dB_{ct} + (e^{Z_{ct}} - 1) S_t dN_{ct} + (e^{Z_{cq,t}} - 1) S_t dN_{cq,t}. \quad (\text{A.1})$$

Consider an agent who allocates wealth between S and the risk-free asset. Let α_t be the fraction of wealth in the risky asset S_t , and let c_t be the agent's consumption. The

wealth process is then given by

$$dW_t = (W_t \alpha_t (\mu - r_t + l^{-1}) + W_t r_t - c_t) dt + W_t \alpha_t \sigma dB_{ct} \\ + \alpha_t W_t ((e^{Z_{ct}} - 1) S_t dN_{ct} + (e^{Z_{cq,t}} - 1) S_t dN_{cq,t}),$$

where r_t denotes the instantaneous risk-free rate. Optimal consumption and portfolio choices must satisfy the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\sup_{\alpha_t, C_t} \left\{ J_W (W_t \alpha_t (\mu - r_t + l^{-1}) + W_t r_t - c_t) + \kappa_{\lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + \kappa_{\lambda_{cq}} (\bar{\lambda}_{cq} - \lambda_{cq,t}) \right. \\ \left. + \frac{1}{2} J_{WW} W_t^2 \alpha_t^2 \sigma^2 + \frac{1}{2} (J_{\lambda_c \lambda_c} \sigma_{\lambda_c}^2 \lambda_{ct} + J_{\lambda_{cq} \lambda_{cq}} \sigma_{\lambda_{cq}}^2 \lambda_{cq,t}) \right. \\ \left. + \lambda_{ct} E_{\nu_c} [J(W_t (1 + \alpha_t (e^{Z_{ct}} - 1)), \lambda_t) - J(W_t, \lambda_t)] \right. \\ \left. + \lambda_{cq,t} E_{\nu_{cq}} [J(W_t (1 + \alpha_t (e^{Z_{cq,t}} - 1)), \lambda_t) - J(W_t, \lambda_t)] + f(c_t, V_t) \right\} = 0, \quad (\text{A.2})$$

where J_n denotes the first derivative of J with respect to variable n , for n equal to λ_i or W , and J_{nm} denotes the second derivative of J with respect to n and m .

In equilibrium, $\alpha_t = 1$ and $c_t = W_t l^{-1}$. Substituting these policy functions into (A.2) implies

$$J_W W_t \mu + J_{\lambda_c} \kappa_{\lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + J_{\lambda_{cq}} \kappa_{\lambda_{cq}} (\bar{\lambda}_{cq} - \lambda_{cq,t}) + \frac{1}{2} J_{WW} W_t^2 \sigma^2 \\ + \frac{1}{2} (J_{\lambda_c \lambda_c} \sigma_{\lambda_c}^2 \lambda_{ct} + J_{\lambda_{cq} \lambda_{cq}} \sigma_{\lambda_{cq}}^2 \lambda_{cq,t}) + \lambda_{ct} E_{\nu_c} [J(W_t e^{Z_{ct}}, \lambda_t) - J(W_t, \lambda_t)] \\ + \lambda_{cq,t} E_{\nu_{cq}} [J(W_t e^{Z_{cq,t}}, \lambda_t) - J(W_t, \lambda_t)] + f(c_t, V_t) = 0. \quad (\text{A.3})$$

By the envelope condition $f_C = J_W$, we obtain $\beta = l^{-1}$. Given the consumption-wealth ratio, it follows that

$$f(c_t, V_t) = f(W_t l^{-1}, J(W_t, \lambda_t)) = \beta W_t^{1-\gamma} \left(\log \beta - \frac{\log I(\lambda_t)}{1-\gamma} \right). \quad (\text{A.4})$$

Substituting (A.4) and (6) into (A.3) and dividing both sides by $W_t^{1-\gamma}I(\lambda_t)$, we find

$$\begin{aligned} \mu + I^{-1}(1-\gamma)^{-1} & \left(I_{\lambda_c} \kappa_{\lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + I_{\lambda_{cq}} \kappa_{\lambda_{cq}} (\bar{\lambda}_{cq} - \lambda_{cq}) \right) - \frac{1}{2} \gamma \sigma^2 \\ & + \frac{1}{2} I^{-1} \left(I_{\lambda_c \lambda_c} \sigma_{\lambda_c}^2 \lambda_{ct} + I_{\lambda_{cq} \lambda_{cq}} \sigma_{\lambda_{cq}}^2 \lambda_{cq,t} \right) \\ & + (1-\gamma)^{-1} \left(\lambda_c E_{\nu_c} [e^{(1-\gamma)Z_c} - 1] + \lambda_{cq} E_{\nu_{cq}} [e^{(1-\gamma)Z_{cq}} - 1] \right) \\ & + \beta \left(\log \beta - \frac{\log I(\lambda_t)}{1-\gamma} \right) = 0, \end{aligned}$$

where I_{λ_j} denotes the first derivative of I with respect to λ_j and $I_{\lambda_j \lambda_j}$ denotes the second derivative for $j \in \{c, cq\}$.

Collecting terms in λ_{jt} results in the following quadratic equation for b_j :

$$\frac{1}{2} \sigma_{\lambda_j}^2 b_j^2 - (\kappa_{\lambda_j} + \beta) b_j + E_{\nu_j} [e^{(1-\gamma)Z_j} - 1],$$

for $j \in \{c, cq\}$, implying

$$b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} \pm \sqrt{\left(\frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} \right)^2 - 2 \frac{E_{\nu_j} [e^{(1-\gamma)Z_j} - 1]}{\sigma_{\lambda_j}^2}},$$

Collecting constant terms results in the following characterization of a in terms of b :

$$a = \frac{1-\gamma}{\beta} \left(\mu - \frac{1}{2} \gamma \sigma^2 \right) + (1-\gamma) \log \beta + \frac{1}{\beta} b^\top (\kappa_\lambda * \bar{\lambda}).$$

Here and in what follows, I use $*$ to denote element-by-element multiplication of vectors of equal dimension. Given the form of $I(\lambda)$, $I_{\lambda_j} = b_j I$ and $I_{\lambda_j \lambda_j} = b_j^2 I$ for $j \in \{c, cq\}$. Because there are no interaction terms, the solution takes the same form as when there is only a single type of jump. As in Wachter (2013, Appendix A.1) we take the negative root of the

corresponding equation for b_j to find:

$$b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} - \sqrt{\left(\frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2}\right)^2 - 2 \frac{E_{\nu_j} [e^{(1-\gamma)Z_j} - 1]}{\sigma_{\lambda_j}^2}}.$$

□

Proof of Corollary 2 Since $\gamma > 1$, if $Z_j < 0$, then the second term in the square root of (9) is positive. Therefore the square root term is positive but less than $\frac{\kappa_j + \beta}{\sigma_j^2}$, and $b_j > 0$. Similarly, if $Z_j > 0$ then the second term in the square root of (9) is negative. Therefore the square root term is positive and greater than $\frac{\kappa_j + \beta}{\sigma_j^2}$, and $b_j < 0$. □

Proof of Corollary 3 The risk-free rate is obtained by taking the derivative of the HJB (A.2) with respect to α_t , evaluating at $\alpha_t = 1$, and setting it equal to 0. The result immediately follows. □

A.3 The state-price density

Duffie and Skiadas (1994) show that the state-price density π_t equals

$$\pi_t = \exp \left\{ \int_0^t f_V(C_s, V_s) ds \right\} f_C(C_t, V_t),$$

where f_C and f_V denote derivatives of f with respect to the first and second argument respectively. Note that the exponential term is deterministic. From equation (4), I obtain

$$f_C(C_t, V_t) = \beta(1 - \gamma) \frac{V}{C}.$$

From the equilibrium condition $V_t = J(\beta^{-1}C_t, \lambda_t)$, together with the form of the value function (6), I get

$$f_C(C_t, V_t) = \beta^\gamma C_t^{-\gamma} I(\lambda_t). \tag{A.5}$$

Applying Ito's Lemma to (A.5) implies

$$\frac{d\pi_t}{\pi_{t-}} = \mu_{\pi t} dt + \sigma_{\pi t} dB_t + (e^{-\gamma Z_{ct}} - 1) dN_{ct} + (e^{-\gamma Z_{cq,t}} - 1) dN_{cq,t}, \quad (\text{A.6})$$

where

$$\sigma_{\pi t} = \left[-\gamma\sigma, 0, 0, b_c\sigma_c\sqrt{\lambda_{ct}}, b_{cq}\sigma_{cq}\sqrt{\lambda_{cq,t}} \right]. \quad (\text{A.7})$$

It also follows from no-arbitrage that

$$\begin{aligned} \mu_{\pi t} &= -r_t - (\lambda_{ct} E_{\nu_c} [e^{-\gamma Z_{ct}} - 1] + \lambda_{cq,t} E_{\nu_{cq}} [e^{-\gamma Z_{cq,t}} - 1]) \\ &= -\beta - \mu + \gamma\sigma^2 - (\lambda_{ct} E_{\nu_c} [e^{(1-\gamma)Z_{ct}} - 1] + \lambda_{ct} E_{\nu_{cq}} [e^{(1-\gamma)Z_{cq,t}} - 1]). \end{aligned} \quad (\text{A.8})$$

From (A.6) we can see that in the event of a disaster, marginal utility (as represented by the state-price density) jumps upward. This implies that investors require compensation for bearing disaster risks. The first element of (A.7) implies that the standard diffusion risk in consumption is priced; more importantly, changes in λ_{jt} are also priced as reflected by the last two elements of (A.7).

The nominal state-price density $\pi^{\$}$ equals

$$\pi_t^{\$} = \frac{\pi_t}{P_t}. \quad (\text{A.9})$$

The nominal state-price density follows

$$\frac{d\pi_t^{\$}}{\pi_{t-}^{\$}} = \mu_{\pi t}^{\$} dt + \sigma_{\pi t}^{\$} dB_t + (e^{-\gamma Z_{ct}} - 1) dN_{ct} + (e^{-\gamma Z_{cq,t}} - 1) dN_{cq,t}, \quad (\text{A.10})$$

where

$$\sigma_{\pi t}^{\$} = \left[-\gamma\sigma, -\sigma_P, 0, b_c\sigma_{\lambda_c}\sqrt{\lambda_{ct}}, b_{cq}\sigma_{\lambda_{cq}}\sqrt{\lambda_{cq,t}} \right], \quad (\text{A.11})$$

and

$$\mu_{\pi t}^{\$} = -\beta - \mu + \gamma\sigma^2 - q_t + \sigma_P^2 - (\lambda_{ct}E_{\nu_c} [e^{(1-\gamma)Z_{ct}} - 1] + \lambda_{ct}E_{\nu_{cq}} [e^{(1-\gamma)Z_{cq,t}} - 1]). \quad (\text{A.12})$$

By comparing (A.11) to (A.7), we can see that the second element is no longer zero. This implies that the diffusion risk in inflation is also priced in the nominal state-price density. By comparing (A.12) to (A.8), we can see that the expected inflation and volatility of realized inflation also affect the drift of the nominal state-price density.

Proof of Corollary 4 It follows from no-arbitrage that

$$\mu_{\pi t}^{\$} = -r_t^{\$} - (\lambda_{ct}E_{\nu_c} [e^{-\gamma Z_{ct}} - 1] + \lambda_{cq,t}E_{\nu_{cq}} [e^{-\gamma Z_{cq,t}} - 1]),$$

where $\mu_{\pi t}^{\$}$ is given by (A.12). Therefore the nominal risk-free rate on a nominal bond, $r_t^{\$}$ is

$$r_t^{\$} = \beta + \mu - \gamma\sigma^2 + q_t - \sigma_P^2 + \lambda_{ct}E_{\nu_c} [e^{-\gamma Z_{ct}} (e^{Z_{ct}} - 1)] + \lambda_{qt}E_{\nu_{cq}} [e^{-\gamma Z_{cq,t}} (e^{Z_{cq,t}} - 1)].$$

□

B Pricing general zero-coupon equity

This section provides the price of a general form of a zero-coupon equity, both in real terms and in nominal terms. The dividend on the aggregate market and the face value on the bond market will be special cases.

B.1 Real assets

First I will consider the price of a real asset. Consider a stream of cash-flow that follows a jump-diffusion process:

$$\frac{dD_t}{D_t^-} = \mu_D dt + \sigma_D dB_t + (e^{\phi_c Z_{ct}} - 1) dN_{ct} + (e^{\phi_{cq} Z_{cq,t}} - 1) dN_{cq,t}. \quad (\text{B.1})$$

Lemma B.1. Let $H(D_t, \lambda_t, \tau)$ denote the time t price of a single future cash-flow at time $s = t + \tau$:

$$H(D_t, \lambda_t, s - t) = E_t \left[\frac{\pi_s}{\pi_t} D_s \right].$$

By Ito's Lemma, we can write

$$\frac{dH(D_t, \lambda_t, \tau)}{H(D_t, \lambda_t, \tau)} = \mu_{H(\tau),t} dt + \sigma_{H(\tau),t}^\top dB_t + \mathcal{J}_c(\pi_t H(D_t, \lambda_t, \tau)) dN_{ct} + \mathcal{J}_{cq}(\pi_t H(D_t, \lambda_t, \tau)) dN_{cq,t}.$$

for a scalar process $\mu_{H(\tau),t}$ and a vector process $\sigma_{H(\tau),t}$. Then, no-arbitrage implies that:

$$\mu_{\pi,t} + \mu_{H(\tau),t} + \sigma_{\pi,t}^\top \sigma_{H(\tau),t} + \frac{1}{\pi_t H_t(\tau)} \lambda_t^\top \bar{\mathcal{J}}(\pi_t H(D_t, \lambda_t, \tau)) = 0. \quad (\text{B.2})$$

Proof No-arbitrage implies that $H(D_s, \lambda_s, 0) = D_s$ and that

$$\pi_t H(D_t, \lambda_t, \tau) = E_t [\pi_s H(D_s, \lambda_s, 0)].$$

To simplify notation, let $H_t = H(D_t, \lambda_t, \tau)$, $\mu_{H,t} = \mu_{H(\tau),t}$, and $\sigma_{H,t} = \sigma_{H(\tau),t}$. It follows from Ito's Lemma that

$$\frac{dH_t}{H_t} = \mu_{H,t} dt + \sigma_{H,t} dB_t + (e^{\phi_c Z_{ct}} - 1) dN_{ct} + (e^{\phi_{cq} Z_{cq,t}} - 1) dN_{cq,t}.$$

Applying Ito's Lemma to $\pi_t H_t$ implies that the product can be written as

$$\begin{aligned} \pi_t H_t = \pi_0 H_0 &+ \int_0^t \pi_s H_s (\mu_{H,s} + \mu_{\pi,s} + \sigma_{\pi,s}^\top \sigma_{H,s}) + \int_0^t \pi_s H_s (\sigma_{H,s} + \sigma_{\pi,s}) dB_s \\ &+ \sum_{0 < s_{ci} \leq t} \left(\pi_{s_{ci}} H_{s_{ci}} - \pi_{s_{ci}^-} H_{s_{ci}^-} \right) + \sum_{0 < s_{cq,i} \leq t} \left(\pi_{s_{cq,i}} H_{s_{cq,i}} - \pi_{s_{cq,i}^-} H_{s_{cq,i}^-} \right), \end{aligned} \quad (\text{B.3})$$

where $s_{ji} = \inf\{s : N_{js} = i\}$ (namely, the time that the i th time type- j jump occurs, where $j \in \{c, cq\}$).

We use (B.3) to derive a no-arbitrage condition. The first step is to compute the

expectation of the jump terms $\sum_{0 < s_{ji} \leq t} \left(\pi_{s_{ji}} H_{s_{ji}} - \pi_{s_{ji}^-} H_{s_{ji}^-} \right)$. The pure diffusion processes are not affected by the jump. Adding and subtracting the jump compensation terms from (B.3) yields:

$$\begin{aligned} \pi_t H_t = & \pi_0 H_0 + \int_0^t \pi_s H_s \left(\mu_{H,s} + \mu_{\pi,s} + \sigma_{\pi,s} \sigma_{H,s}^\top + \frac{1}{\pi_s H_s} (\lambda_c \bar{\mathcal{J}}_c(\pi_s H_s) + \lambda_{cq} \bar{\mathcal{J}}_{cq}(\pi_s H_s)) \right) ds \\ & + \int_0^t \pi_s H_s (\sigma_{H,s} + \sigma_{\pi,s}) dB_s + \sum_{0 < s_{ci} \leq t} \left(\left(\pi_{s_{ci}} H_{s_{ci}} - \pi_{s_{ci}^-} H_{s_{ci}^-} \right) - \int_0^t \pi_s H_s \lambda_c \bar{\mathcal{J}}_c(\pi_s H_s) ds \right) \\ & + \sum_{0 < s_{cq,i} \leq t} \left(\left(\pi_{s_{cq,i}} H_{s_{cq,i}} - \pi_{s_{cq,i}^-} H_{s_{cq,i}^-} \right) - \int_0^t \pi_s H_s \lambda_{cq} \bar{\mathcal{J}}_{cq}(\pi_s H_s) ds \right) \quad (\text{B.4}) \end{aligned}$$

Under mild regularity conditions analogous to those given in Duffie, Pan, and Singleton (2000), the second and the third terms on the right hand side of (B.4) are martingales. Therefore the first term on the right hand side of (B.4) must also be a martingale, and it follows that the integrand of this term must equal zero:

$$\mu_{\pi,t} + \mu_{H(\tau),t} + \sigma_{\pi,t} \sigma_{H(\tau),t}^\top + \frac{1}{\pi_t H_t(\tau)} \lambda_t^\top \bar{\mathcal{J}}(\pi_t H(D_t, \lambda_t, \tau)) = 0.$$

□

Theorem B.2. *The function H takes an exponential form:*

$$H(D_t, \lambda_t, \tau) = D_t \exp \left\{ a_\phi(\tau) + \lambda_t^\top b_{\phi\lambda}(\tau) \right\}, \quad (\text{B.5})$$

where $b_{\phi\lambda} = [b_{\phi\lambda_c}, b_{\phi\lambda_{cq}}]^\top$. Function $b_{\phi\lambda_j}$ for $j \in \{c, cq\}$ solves

$$\frac{db_{\phi\lambda_j}}{d\tau} = \frac{1}{2} \sigma_{\lambda_j}^2 b_{\phi\lambda_j}(\tau)^2 + \left(b_j \sigma_{\lambda_j}^2 - \kappa_{\lambda_j} \right) b_{\phi\lambda_j}(\tau) + E_{\nu_j} \left[e^{(\phi_j - \gamma) Z_{jt}} - e^{(1-\gamma) Z_{jt}} \right] \quad (\text{B.6})$$

and function a_ϕ solves

$$\frac{da_\phi}{d\tau} = \mu_D - \mu - \beta + \gamma \sigma (\sigma - \sigma_D) + b_{\phi\lambda}(\tau)^\top (\kappa_{\lambda_j} * \bar{\lambda}_j). \quad (\text{B.7})$$

The boundary conditions are $a_\phi(0) = b_{\phi\lambda_c}(0) = b_{\phi\lambda_{cq}}(0) = 0$.

Proof See proof of Theorem B.4. □

B.2 Nominal asset

Similar no-arbitrage conditions can be derived for nominally denominated assets. Suppose cash-flow that follows:

$$\frac{dD_t^\$}{D_{t-}^\$} = \mu_{D^\$} dt + \sigma_{D^\$} dB_t + (e^{\phi_c^\$ Z_{ct}} - 1) dN_{ct} + (e^{\phi_{cq}^\$ Z_{cq,t}} - 1) dN_{cq,t}.$$

Lemma B.3. Let $H^\$(D_t^\$, q_t, \lambda_t, \tau)$ denote the time t price of a single future dividend payment at time $t + \tau$:

$$H^\$(D_t^\$, q_t, \lambda_t, s - t) = E_t \left[\frac{\pi_s^\$}{\pi_t^\$} D_s^\$ \right].$$

By Ito's Lemma, we can write

$$\begin{aligned} \frac{dH^\$(D_t^\$, q_t, \lambda_t, \tau)}{H^\$(D_t^\$, q_t, \lambda_t, \tau)} &= \mu_{H^\$(\tau),t} dt + \sigma_{H^\$(\tau),t}^\top dB_t + \mathcal{J}_c(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau)) dN_{ct} \\ &\quad + \mathcal{J}_{cq}(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau)) dN_{cq,t} + \mathcal{J}_q(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau)) dN_{qt}. \end{aligned}$$

for a scalar process $\mu_{H^\$(\tau),t}$ and a vector process $\sigma_{H^\$(\tau),t}$. Then, no-arbitrage implies that:

$$\begin{aligned} \mu_{\pi^\$,t} + \mu_{H^\$(\tau),t} + \sigma_{\pi^\$,t}^\top \sigma_{H^\$(\tau),t} + \frac{1}{\pi_t^\$ H_t^\$(\tau)} \left(\lambda_{ct} \bar{\mathcal{J}}_c(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau)) \right. \\ \left. + \lambda_{cq,t} (\bar{\mathcal{J}}_{cq}(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau)) + \bar{\mathcal{J}}_q(\pi_t^\$ H^\$(D_t^\$, q_t, \lambda_t, \tau))) \right) = 0, \quad (\text{B.8}) \end{aligned}$$

Proof See proof of Lemma B.1. □

Theorem B.4. The function $H^\$$ takes an exponential form:

$$H^\$(D_t^\$, q_t, \lambda_t, \tau) = D_t^\$ \exp \{ a_{\phi^\$}(\tau) + b_{\phi^\$q}(\tau) q_t + b_{\phi^\$\lambda}(\tau)^\top \lambda_t \}, \quad (\text{B.9})$$

where $b_{\phi^s \lambda} = [b_{\phi^s \lambda_c}, b_{\phi^s \lambda_{cq}}]$. Function $b_{\phi^s q}$ solves

$$\frac{db_{\phi^s q}}{d\tau} = -\kappa_q b_{\phi^s q}(\tau) - 1; \quad (\text{B.10})$$

function $b_{\phi^s \lambda_c}$ solves

$$\frac{db_{\phi^s \lambda_c}}{d\tau} = \frac{1}{2} \sigma_{\lambda_c}^2 b_{L\lambda_c}(\tau)^2 + (b_c \sigma_{\lambda_c}^2 - \kappa_{\lambda_c}) b_{\phi^s \lambda_c}(\tau) + E_{\nu_c} \left[e^{(\phi_c^s - \gamma) Z_{ct}} - e^{(1-\gamma) Z_{ct}} \right]; \quad (\text{B.11})$$

function $b_{\phi^s \lambda_{cq}}$ solves

$$\begin{aligned} \frac{db_{\phi^s \lambda_{cq}}}{d\tau} = & \frac{1}{2} \sigma_{\lambda_{cq}}^2 b_{\phi^s \lambda_{cq}}(\tau)^2 + (b_{cq} \sigma_{\lambda_{cq}}^2 - \kappa_{\lambda_{cq}}) b_{\phi^s \lambda_{cq}}(\tau) \\ & + E_{\nu_{cq}} \left[e^{(\phi_{cq}^s - (\gamma + b_{\phi^s q}(\tau))) Z_{cq,t}} - e^{(1-\gamma) Z_{cq,t}} \right] + E_{\nu_q} \left[e^{-b_{\phi^s q}(\tau) Z_{qt}} - 1 \right]; \end{aligned} \quad (\text{B.12})$$

and function a_L solves

$$\frac{da_{\phi^s}}{d\tau} = \mu_D - \beta - \mu + \gamma \sigma (\sigma - \sigma_D) + \sigma_P^2 + \frac{1}{2} \sigma_q^2 b_{\phi^s q}(\tau)^2 + b_{\phi^s q}(\tau) \kappa_q \bar{q} + b_{\phi^s \lambda}(\tau)^\top (\kappa_\lambda * \bar{\lambda}). \quad (\text{B.13})$$

The boundary conditions are $a_{\phi^s}(0) = b_{\phi^s q}(0) = b_{\phi^s \lambda_c}(0) = b_{\phi^s \lambda_{cq}}(0) = 0$.

Proof It follows from Ito's Lemma that

$$\frac{dH_t^s}{H_t^{s-}} = \mu_{H^s,t} dt + \sigma_{H^s,t} dB_t + \frac{1}{H_t^{s-}} \left(\mathcal{J}_c(H_t^s) + \mathcal{J}_{cq}(H_t^s) + \mathcal{J}_q(H_t^s) \right),$$

where μ_{H^s} and σ_{H^s} are given by

$$\begin{aligned}
\mu_{H^s,t} &= \frac{1}{H^s} \left(\frac{\partial H^s}{\partial q} (\bar{q} - q_t) + \frac{\partial H^s}{\partial \lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + \frac{\partial H^s}{\partial \lambda_{cq}} (\bar{\lambda}_c - \lambda_{cq,t}) - \frac{\partial H^s}{\partial \tau} \right. \\
&\quad \left. + \frac{1}{2} \frac{\partial^2 H^s}{\partial q_j^2} \sigma_q^2 + \frac{1}{2} \left(\frac{\partial^2 H^s}{\partial \lambda_c^2} \sigma_{\lambda_c}^2 + \frac{\partial^2 H^s}{\partial \lambda_c^2} \sigma_{\lambda_c}^2 \right) \right) \\
&= b_{\phi^s q}(\tau) \kappa_q (\bar{q} - q_t) + b_{\phi^s \lambda_c}(\tau) \kappa_{\lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + b_{\phi^s \lambda_{cq}}(\tau) \kappa_{\lambda_{cq}} (\bar{\lambda}_{cq} - \lambda_{cq,t}) \\
&\quad + \frac{1}{2} b_{\phi^s q}(\tau)^2 \sigma_q^2 + \frac{1}{2} \left(b_{\phi^s \lambda_c}(\tau)^2 \sigma_{\lambda_c}^2 \lambda_{ct} + b_{\phi^s \lambda_{cq}}(\tau)^2 \sigma_{\lambda_{cq}}^2 \lambda_{cq,t} \right) \\
&\quad - \left(\frac{da_{\phi^s}}{d\tau} + \frac{db_{\phi^s q}}{d\tau} q_t + \sum_j \frac{db_{\phi^s \lambda_j}}{d\tau} \lambda_{jt} \right), \quad (\text{B.14})
\end{aligned}$$

and

$$\begin{aligned}
\sigma_{H^s,t} &= \frac{1}{L} \left(\frac{\partial H^s}{\partial q_t} [0, 0, \sigma_q \sqrt{q_t}, 0, 0] + \frac{\partial H^s}{\partial \lambda_c} [0, 0, 0, \sigma_{\lambda_c} \sqrt{\lambda_{ct}}, 0] + \frac{\partial H^s}{\partial \lambda_{cq}} [0, 0, 0, 0, \sigma_{\lambda_{cq}} \sqrt{\lambda_{cq,t}}] \right) \\
&= [0, 0, b_{\phi^s q}(\tau) \sigma_q \sqrt{q_t}, b_{\phi^s \lambda_c}(\tau) \sigma_{\lambda_c} \sqrt{\lambda_{ct}}, b_{\phi^s \lambda_{cq}}(\tau) \sigma_{\lambda_{cq}} \sqrt{\lambda_{cq,t}}]. \quad (\text{B.15})
\end{aligned}$$

Furthermore,

$$\frac{\bar{\mathcal{J}}_c(\pi_t^s H_t^s)}{\pi_t^s H_t^s} = E_{\nu_c} \left[e^{(\phi_c^s - \gamma) Z_{ct}} - 1 \right], \quad (\text{B.16})$$

$$\frac{\bar{\mathcal{J}}_{cq}(\pi_t^s H_t^s)}{\pi_t^s H_t^s} = E_{\nu_{cq}} \left[e^{(\phi_{cq}^s - (\gamma + b_{\phi^s q}(\tau))) Z_{cq,t}} - 1 \right], \quad (\text{B.17})$$

and

$$\frac{\bar{\mathcal{J}}_q(\pi_t^s H_t^s)}{\pi_t^s H_t^s} = E_{\nu_q} \left[e^{-b_{\phi^s q}(\tau) Z_{qt}} - 1 \right]. \quad (\text{B.18})$$

Recall that $\lambda_q = \lambda_{cq}$. Substituting (B.14) – (B.17) along with (A.11) and (A.12) into the no-arbitrage condition (B.8) implies that functions a_{ϕ^s} , $b_{\phi^s q}$, $b_{\phi^s \lambda_c}$, and $b_{\phi^s \lambda_{cq}}$ solve the

following ordinary differential equation:

$$\begin{aligned}
& b_{\phi^s q}(\tau)\kappa_q(\bar{q} - q_t) + b_{\phi^s \lambda_c}(\tau)\kappa_{\lambda_c}(\bar{\lambda}_c - \lambda_{ct}) + b_{\phi^s \lambda_{cq}}(\tau)\kappa_{\lambda_{cq}}(\bar{\lambda}_{cq} - \lambda_{cq,t}) \\
& + \frac{1}{2}b_{\phi^s q}(\tau)^2\sigma_q^2 + \frac{1}{2}\left(b_{\phi^s \lambda_c}(\tau)^2\sigma_{\lambda_c}^2\lambda_{ct} + b_{\phi^s \lambda_c}(\tau)^2\sigma_{\lambda_c}^2\lambda_{ct}\right) - \beta - \mu + \gamma\sigma^2 - q_t + \sigma_P^2 \\
& + b_{\phi^s \lambda_c}(\tau)b_j\sigma_{\lambda_c}^2\lambda_{ct} + b_{\phi^s \lambda_{cq}}(\tau)b_j\sigma_{\lambda_{cq}}^2\lambda_{cq,t} + \lambda_{ct}E_{\nu_c}\left[e^{(\phi_c^s - \gamma)Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right] \\
& + \lambda_{cq,t}\left(E_{\nu_{cq}}\left[e^{(\phi_{cq}^s - (\gamma + b_{\phi^s q}(\tau)))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] + E_{\nu_q}\left[e^{-b_{\phi^s q}(\tau)Z_{qt}} - 1\right]\right) \\
& - \left(\frac{da_{\phi^s}}{d\tau} + \frac{db_{\phi^s q}}{d\tau}q_t + \frac{db_{\phi^s \lambda_c}}{d\tau}\lambda_{ct} + \frac{db_{\phi^s \lambda_{cq}}}{d\tau}\lambda_{cq,t}\right) = 0. \quad (\text{B.19})
\end{aligned}$$

Collecting q_t terms results in the following ordinary differential equation:

$$\frac{db_{\phi^s q}}{d\tau} = -\kappa_q b_{\phi^s q}(\tau) - 1;$$

collecting terms multiplying λ_c results in the following ordinary differential equation for

$b_{\phi^s \lambda_c}$

$$\frac{db_{\phi^s \lambda_c}}{d\tau} = \frac{1}{2}\sigma_{\lambda_c}^2 b_{L\lambda_c}(\tau)^2 + (b_c\sigma_{\lambda_c}^2 - \kappa_{\lambda_c})b_{\phi^s \lambda_c}(\tau) + E_{\nu_c}\left[e^{(\phi_c^s - \gamma)Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right];$$

collecting terms multiplying λ_{cq} results in the following ordinary differential equation for

$b_{\phi^s \lambda_{cq}}$

$$\begin{aligned}
\frac{db_{\phi^s \lambda_{cq}}}{d\tau} &= \frac{1}{2}\sigma_{\lambda_{cq}}^2 b_{\phi^s \lambda_{cq}}(\tau)^2 + \left(b_{cq}\sigma_{\lambda_{cq}}^2 - \kappa_{\lambda_{cq}}\right)b_{\phi^s \lambda_{cq}}(\tau) \\
&+ E_{\nu_{cq}}\left[e^{(\phi_{cq}^s - (\gamma + b_{\phi^s q}(\tau)))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] + E_{\nu_q}\left[e^{-b_{\phi^s q}(\tau)Z_{qt}} - 1\right];
\end{aligned}$$

and collecting constant terms results in the following ordinary differential equation for a_L :

$$\frac{da_{\phi^s}}{d\tau} = \mu_D - \beta - \mu + \gamma\sigma(\sigma - \sigma_D) + \sigma_P^2 + \frac{1}{2}\sigma_q^2 b_{\phi^s q}(\tau)^2 + b_{\phi^s q}(\tau)\kappa_q\bar{q} + b_{\phi^s \lambda}(\tau)^\top(\kappa_\lambda * \bar{\lambda}).$$

The boundary conditions are $a_{\phi^s}(0) = b_{\phi^s q}(0) = b_{\phi^s \lambda_c}(0) = b_{\phi^s \lambda_{cq}}(0) = 0$. \square

C Nominal bond pricing

Proof of Corollary 7 By the no-arbitrage condition (B.8) and the definition of $\mu_{\pi^{\$}}$ (A.12), we can rewrite the premium in population (22) as

$$\begin{aligned} r_t^{\$, (\tau)} - r_t^{\$} &= -\sigma_{\pi^{\$, t}} \sigma_{L, t}^{\top} - \lambda_{ct} \left(\frac{\bar{\mathcal{J}}_c(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} - \frac{\bar{\mathcal{J}}_c(\pi_t^{\$})}{\pi_t^{\$}} - \frac{\bar{\mathcal{J}}_c(L_t^{\$})}{L_t^{\$}} \right) \\ &\quad - \lambda_{cq, t} \left(\frac{\bar{\mathcal{J}}_{cq}(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} - \frac{\bar{\mathcal{J}}_{cq}(\pi_t^{\$})}{\pi_t^{\$}} - \frac{\bar{\mathcal{J}}_{cq}(L_t^{\$})}{L_t^{\$}} \right) - \lambda_{qt} \left(\frac{\bar{\mathcal{J}}_q(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} - \frac{\bar{\mathcal{J}}_q(\pi_t^{\$})}{\pi_t^{\$}} - \frac{\bar{\mathcal{J}}_q(L_t^{\$})}{L_t^{\$}} \right). \end{aligned}$$

From (A.10), we know that for $j \in \{c, cq\}$,

$$\frac{\bar{\mathcal{J}}_j(\pi_t^{\$})}{\pi_t^{\$}} = E_{\nu_j} [e^{-\gamma Z_{jt}} - 1].$$

Recall that N_q type of jumps (inflation spike) does not affect $\pi^{\$}$, therefore, $\frac{\bar{\mathcal{J}}_q(\pi_t^{\$})}{\pi_t^{\$}} = 0$ and $\frac{\bar{\mathcal{J}}_q(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} = \frac{\bar{\mathcal{J}}_q(L_t^{\$})}{L_t^{\$}}$. From (B.16) – (B.17), together with $\phi_c^{\$} = \phi_{cq}^{\$} = 0$, we know that

$$\frac{\bar{\mathcal{J}}_c(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} = E_{\nu_c} [e^{-\gamma Z_{ct}} - 1], \quad \text{and} \quad \frac{\bar{\mathcal{J}}_{cq}(\pi_t^{\$} L_t^{\$})}{\pi_t^{\$} L_t^{\$}} = E_{\nu_{cq}} [e^{-(\gamma + b_{Lq}^{\$(\tau)}) Z_{cq, t}} - 1].$$

Furthermore,

$$\frac{\bar{\mathcal{J}}_c(L_t^{\$})}{L_t^{\$}} = 0, \quad \text{and} \quad \frac{\bar{\mathcal{J}}_{cq}(L_t^{\$})}{L_t^{\$}} = E_{\nu_{cq}} [e^{-b_{Lq}^{\$(\tau)} Z_{cq, t}} - 1].$$

Together with (A.11) and (B.15), we obtain:

$$r_t^{\$, (\tau)} - r_t^{\$} = -\lambda_t^{\top} (b_{L^{\$}\lambda}(\tau) * b * \sigma_{\lambda}^2) + \lambda_{cq} E_{\nu_{cq}} [(e^{-\gamma Z_{cq, t}} - 1)(1 - e^{-b_{Lq}^{\$(\tau)} Z_{cq, t}})].$$

□

References

- Abel, Andrew, 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics* 43, 3–33.
- Ang, Andrew, Sen Dong, and Monika Piazzesi, 2007, No-Arbitrage Taylor Rules, Working paper.
- Ang, Andrew, and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50, 745–787.
- Bakshi, Gurdip S., and Zhiwu Chen, 1996, Inflation, asset prices, and the term structure of interest rates in monetary economies, *Review of Financial Studies* 9, 241–275.
- Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An Empirical Evaluation of the Long-Run Risks Model for Asset Prices, *Critical Finance Review* 1, 183–221.
- Bansal, Ravi, and Ivan Shaliastovich, 2013, A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets, *Review of Financial Studies* 26, 1–33.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long-run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Barro, Robert J., 2006, Rare disasters and asset markets in the twentieth century, *The Quarterly Journal of Economics* 121, 823–866.
- Barro, Robert J., and Tao Jin, 2011, On the size distribution of macroeconomic disasters, working paper, Harvard University.
- Barro, Robert J., and Jose F. Ursua, 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* pp. 255–350.
- Beeler, Jason, and John Y. Campbell, 2012, The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment, *Critical Finance Review* 1, 141–182.

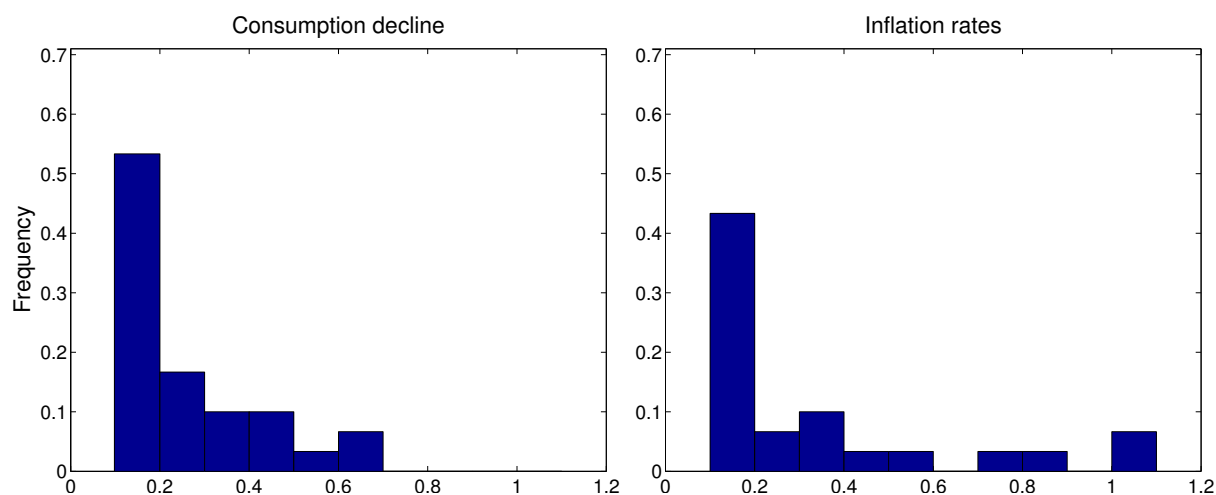
- Bekaert, Geert, Eric Engstrom, and Steven R. Grenadier, 2010, Stock and bond returns with Moody Investors, *Journal of Empirical Finance* 17, 867–894.
- Bekaert, Geert, Robert J. Hodrick, and David A. Marshall, 2001, Peso problem explanations for term structure anomalies, *Journal of Monetary Economics* 48, 241–270.
- Bikbov, Ruslan, and Mikhail Chernov, 2010, No-Arbitrage Macroeconomic Determinants of the Yield Curve, *Journal of Econometrics* 159, 166–182.
- Buraschi, Andrea, and Alexei Jiltsov, 2007, Habit formation and macroeconomic models of the term structure of interest rates, *Journal of Finance* 62, 3009–3063.
- Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.
- Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.
- Campbell, John Y., and Robert J. Shiller, 1991, Yield spreads and interest rate movements: A bird’s eye view, *Review of Economic Studies* 58, 495–514.
- Campbell, John Y., and Luis Viceira, 2001, Who should buy long-term bonds?, *American Economic Review* 91, 99–127.
- Cochrane, John H., 1992, Explaining the variance of price-dividend ratios, *Review of Financial Studies* 5, 243–280.
- Cochrane, John H., and Monika Piazzesi, 2005, Bond risk premia, *American Economic Review* 95, 138–160.
- Collin-Dufresne, Pierre, and Robert S. Goldstein, 2002, Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility, *Journal of Finance* 57, 1685–1730.

- Collin-Dufresne, Pierre, Robert S. Goldstein, and Christopher S. Jones, 2009, Can interest rate volatility be extracted from the cross section of bond yields?, *Journal of Financial Economics* 94, 47–66.
- Cox, John C., Jonathan C. Ingersoll, and Stephen A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–408.
- Dai, Qiang, and Kenneth Singleton, 2002, Expectations puzzles, time-varying risk premia, and affine models of the term structure, *Journal of Financial Economics* 63, 415–442.
- Duffee, Gregory R., 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, 369–443.
- Duffee, Gregory R., 2006, Term structure estimation without using latent factors, *Journal of Financial Economics* 79, 507–536.
- Duffee, Gregory R., 2010, Sharpe ratios in term structure models, Working paper.
- Duffee, Gregory R., 2012, Bond pricing and the macroeconomy, forthcoming, *Handbook of the Economics of Finance*.
- Duffie, Darrell, and Larry G. Epstein, 1992, Asset pricing with stochastic differential utility, *Review of Financial Studies* 5, 411–436.
- Duffie, Darrell, Jun Pan, and Kenneth Singleton, 2000, Transform analysis and asset pricing for affine jump-diffusions, *Econometrica* 68, 1343–1376.
- Duffie, Darrell, and Costis Skiadas, 1994, Continuous-time asset pricing: A utility gradient approach, *Journal of Mathematical Economics* 23, 107–132.
- Ehling, Paul, Michael F. Gallmeyer, Christian Heyerdahl-Larsen, and Philipp K. Illeditsch, 2012, Beliefs about Inflation and the Term Structure of Interest Rates, Working paper.

- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Fama, Eugene F., and Robert R. Bliss, 1987, The information in long-maturity forward rates, *American Economic Review* 77, 680–692.
- Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23–49.
- Fama, Eugene F., and Kenneth R. French, 2001, Disappearing dividends: changing firm characteristics or lower propensity to pay?, *Journal of Financial Economics* 60, 3 – 43.
- Gabaix, Xavier, 2012, An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.
- Gourio, Francois, 2008, Time-series predictability in the disaster model, *Finance Research Letters* 5, 191–203.
- Keim, Donald B., and Robert F. Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357–390.
- Lettau, Martin, and Jessica A. Wachter, 2011, The term structures of equity and interest rates, *Journal of Financial Economics* 101, 90–113.
- Litterman, Robert, and Jose A. Scheinkman, 1991, Common Factors Affecting Bond Returns, *Journal of Fixed Income* pp. 54–61.
- Longstaff, Francis A., and Monika Piazzesi, 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* 74, 401–421.
- Piazzesi, Monika, and Martin Schneider, 2006, Equilibrium yield curves, *NBER Macroeconomics Annual* 21, 389–442.

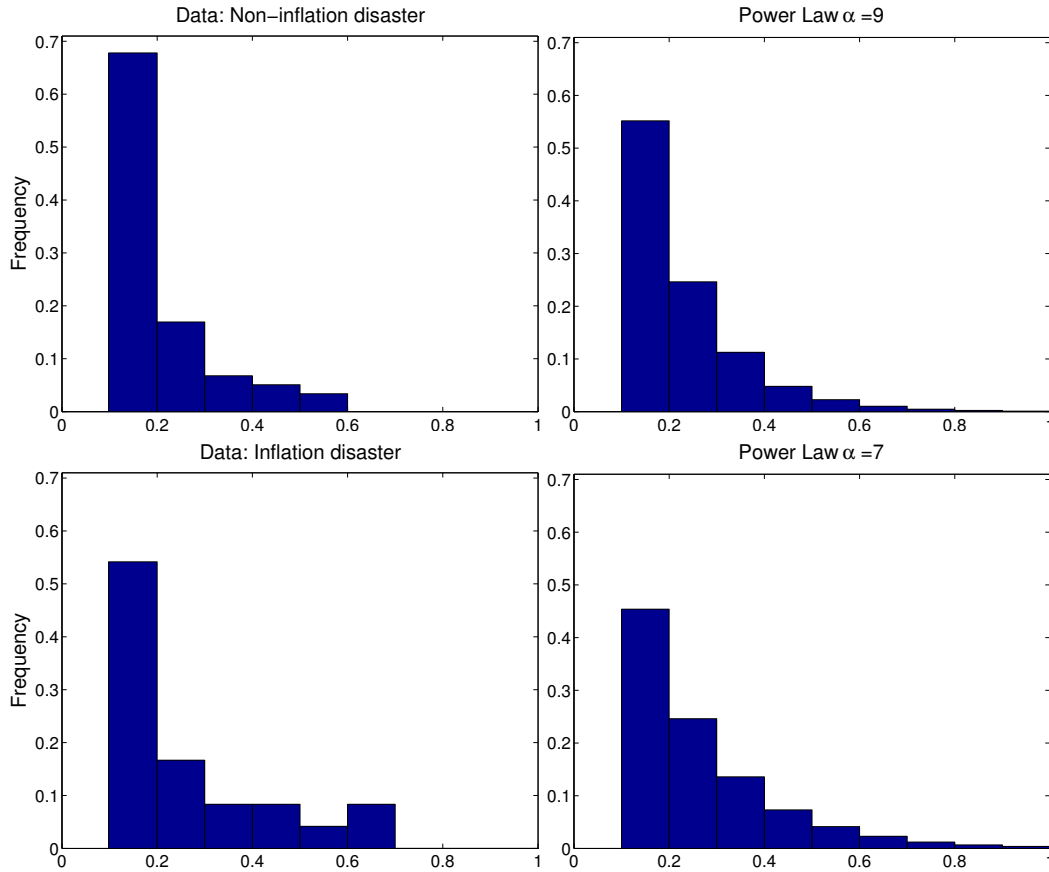
- Rietz, Thomas A., 1988, The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.
- Rudebusch, Glenn D., and Eric T. Swanson, 2008, Examining the Bond Premium Puzzle with a DSGE Model, *Journal of Monetary Economics* 55, 111–126.
- Rudebusch, Glenn D., and Eric T. Swanson, 2012, The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks, *American Economic Journal: Macroeconomics* 4, 105–143.
- Rudebusch, Glenn D., and Tao Wu, 2008, A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy, *Economic Journal* 118, 906–926.
- Stambaugh, Robert F., 1988, The information in forward rates: Implications for models of the term structure, *Journal of Financial Economics* 21, 41–70.
- Wachter, Jessica A., 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365–399.
- Wachter, Jessica A., 2013, Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?, *Journal of Finance* 3, 987–1035.
- Weil, Philippe, 1990, Nonexpected utility in macroeconomics, *Quarterly Journal of Economics* 105, 29–42.

Figure 1: Inflation disasters: Distribution of consumption declines and inflation rates



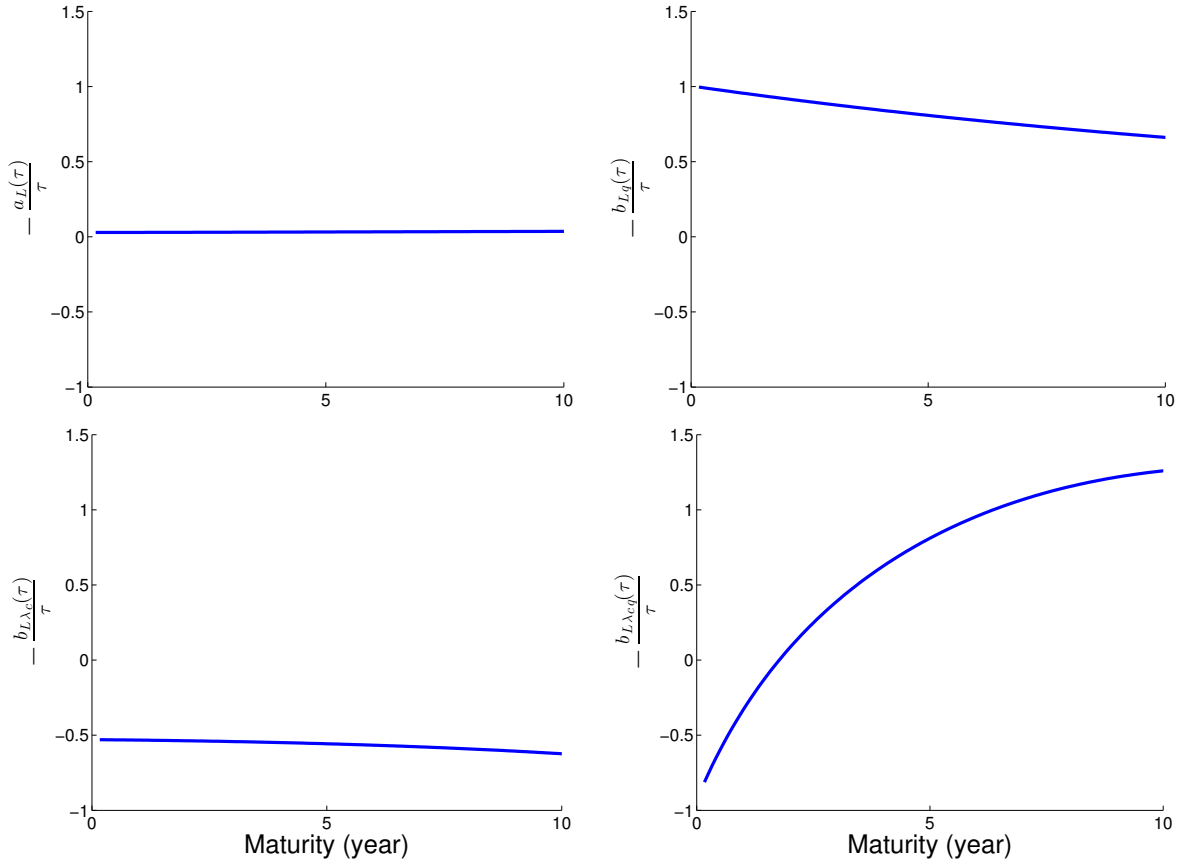
Notes: Histograms show the distribution of large consumption declines (peak-to-trough measure) and high inflation (average annual inflation rate) in periods where large consumption declines and high inflation co-occur. These figures exclude eight events in which average annual inflation rates exceeded 100%. Data from Barro and Ursua (2008).

Figure 2: Data vs. model consumption declines



Notes: This figure plots the distributions of large consumption declines in the data and the power law distribution used in the model. The top-left panel plots the distributions of large consumption declines that do not co-occur with high inflation and the top-right panel plots the power law distribution with parameter 10. The bottom-left panel plots the distributions of large consumption declines that co-occur with high inflation and the bottom-right panel plots the power law distribution with parameter 8. Data from Barro and Ursua (2008).

Figure 3: Solution for the nominal bond yield

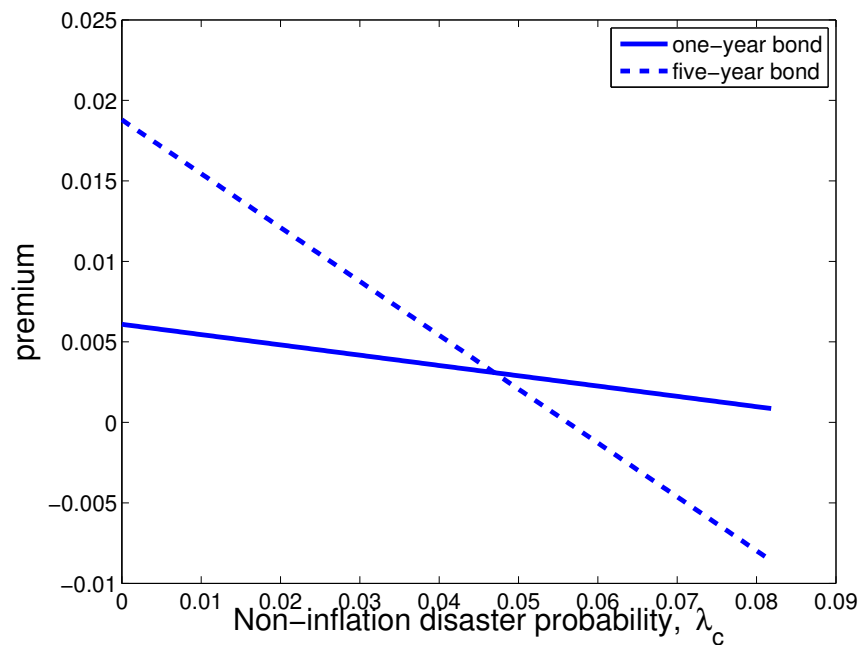


Notes: The nominal yield of a bond with maturity τ is

$$y_t^{\$,(\tau)} = -\frac{1}{\tau} (a_L(\tau) + b_{Lq}(\tau)q_t + b_{L\lambda}(\tau)^\top \lambda_t).$$

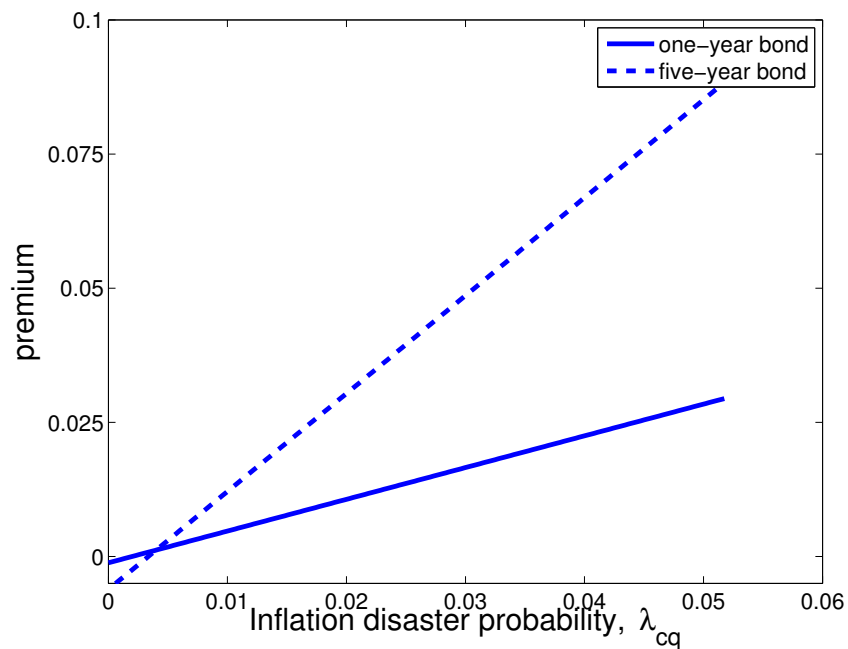
The top-left panel plots the constant term, the top-right panel plots the coefficient multiplying q_t (expected inflation), the bottom-left panel plots the coefficient multiplying λ_c (non-inflation disaster probability), and the bottom right panel plots the coefficient multiplying λ_{cq} (inflation disaster probability). All are plotted as functions of years to maturity (τ).

Figure 4: Risk premiums as a function of non-inflation disaster probability



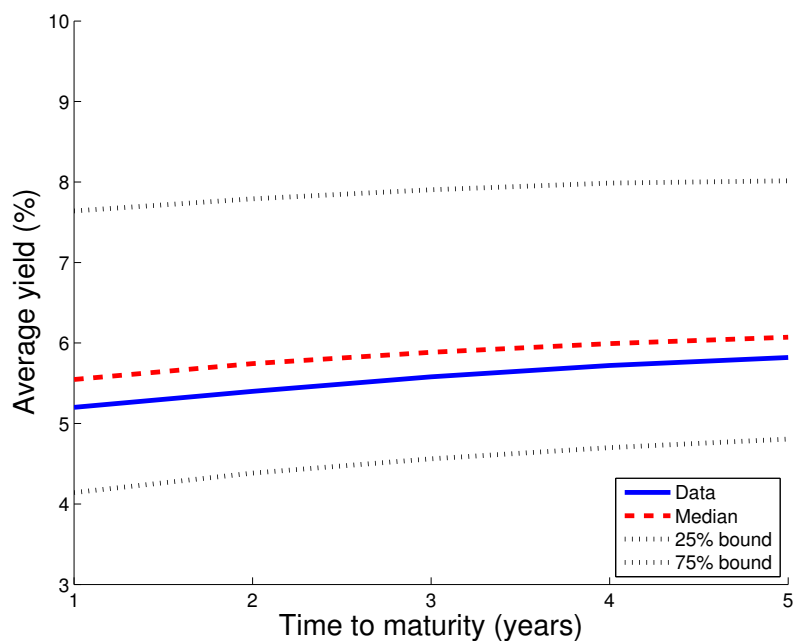
Notes: This figure shows the instantaneous expected nominal return on a one-year nominal zero coupon bond above the nominal risk-free rate (solid line) and the analogous premium for the five-year nominal zero coupon bond (dashed line). Premiums are shown as a function of the non-inflation disaster probability, λ_1 , while λ_2 is fixed at its mean of 1.03%. Premiums are in annual terms.

Figure 5: Risk premiums as a function of inflation disaster probability



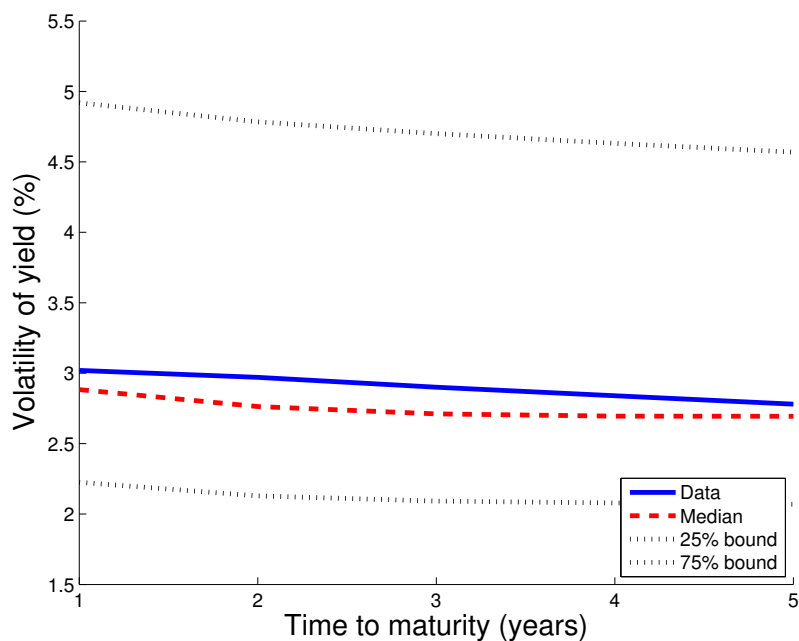
Notes: This figure shows the instantaneous expected nominal return on a one-year nominal zero coupon bond above the nominal risk-free rate (solid line) and the analogous premium for the five-year nominal zero coupon bond (dashed line). Premiums are shown as a function of the disaster probability, λ_2 , while λ_1 is fixed at its mean of 1.83%. Premiums are in annual terms.

Figure 6: Average bond yield



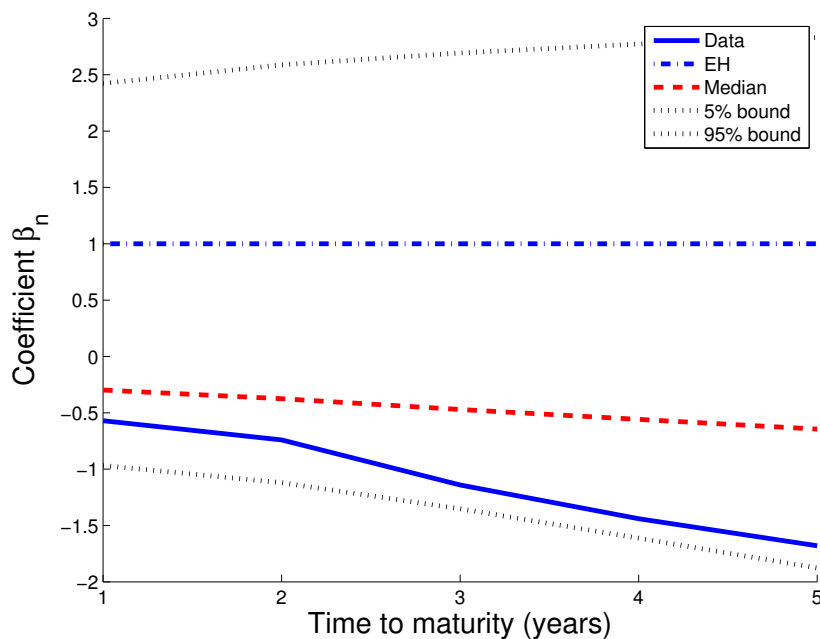
Notes: This figure plots the data and model-implied average nominal bond yield as a function of years to maturity. The solid line plots the average nominal bond yields in the data. The dashed line plots the median average bond yields in the small sample containing no consumption disasters, and the dotted lines plot the 25% and 75% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using the Fama-Bliss dataset from CRSP. All yields are in annual terms.

Figure 7: Volatility of bond yield



Notes: This figure plots the data and model-implied volatility of nominal bond yield as a function of years to maturity. The solid line plots the volatility of nominal bond yields in the data. The dashed line plots the median volatility of bond yields in the small-samples containing no consumption disasters, and the dotted lines plot the 25% and 75% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using the Fama-Bliss dataset from CRSP. All yields are in annual terms.

Figure 8: Campbell-Shiller long rate regression

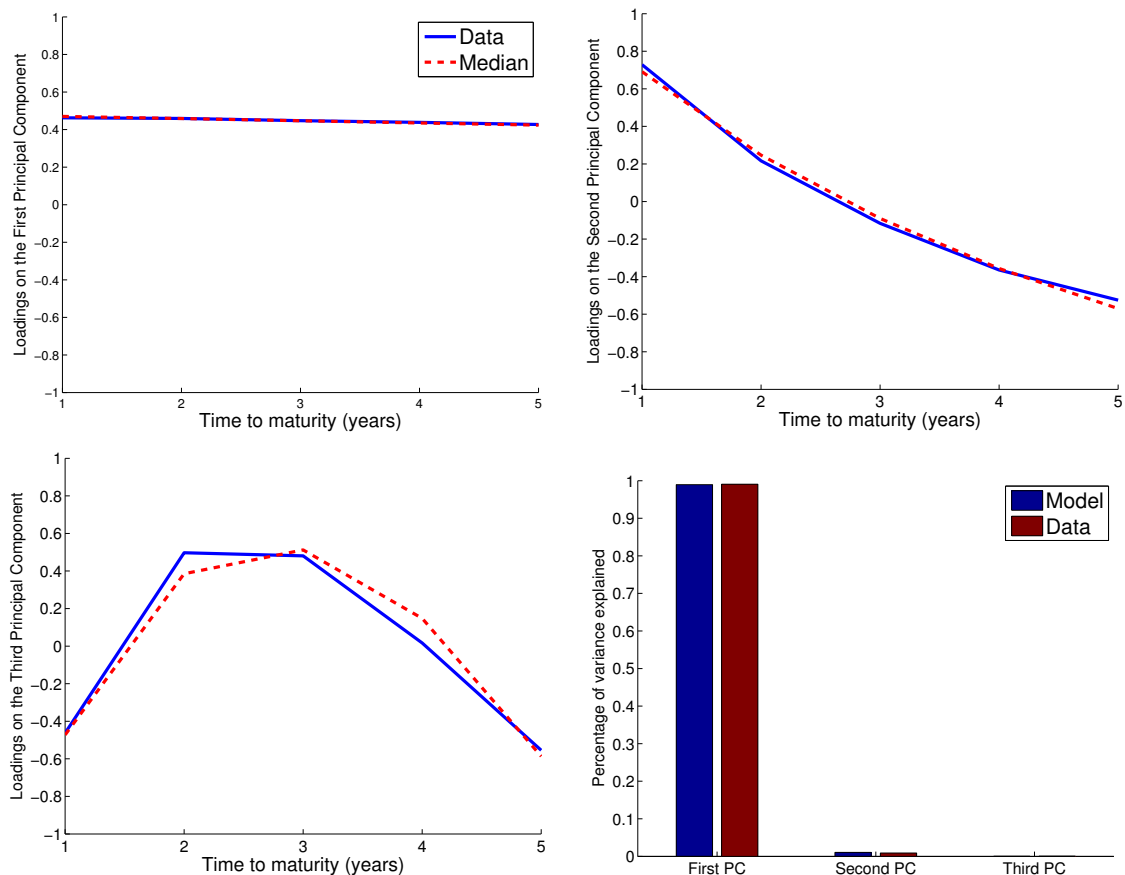


Notes: This figure reports the coefficients of the Campbell-Shiller regression.

$$y_{t+h}^{\$, (n-h)} - y_t^{\$, (n)} = \text{constant} + \beta_n \frac{1}{n-h} \left(y_t^{\$, (n)} - y_t^{\$, (h)} \right) + \text{error},$$

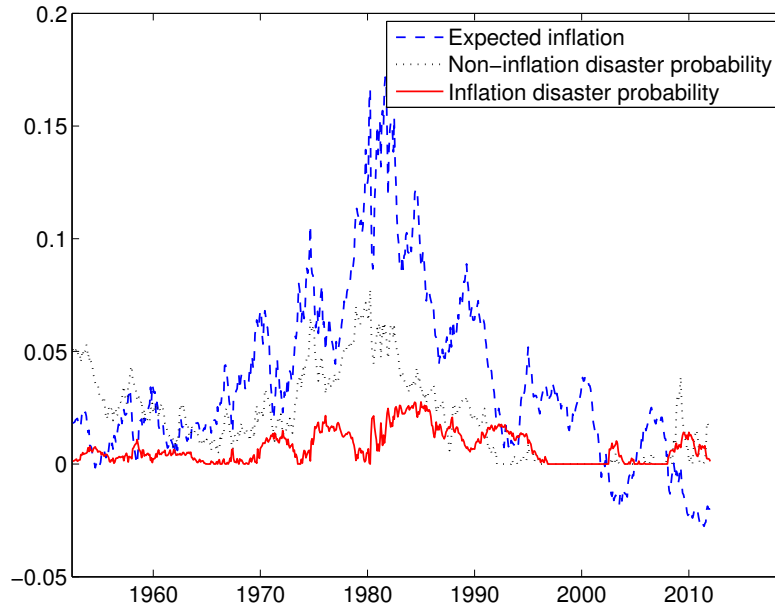
where $h = 0.25$. The solid line plots the coefficients in the data. The dash-dotted line plots the coefficients under the expectation hypothesis. The dashed line plots the median value of the coefficients in the small-samples containing no consumption disasters, and the dotted lines plot the 5% and 95% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using Fama-Bliss dataset from the CRSP.

Figure 9: Principal component analysis



Notes: This figure plots the results from the principal component analysis. I report the median values from the subset of small-sample simulations that do not contain any disasters. The top-left panel plots the loadings on the first principal component, the top-right panel plots the loadings on the second principal component, and the bottom-left panel plots the loadings on the third principal component. The bottom-right panel shows the percentage of variance explained by each of the principal components. Data are at monthly frequency from June 1952 to December 2011.

Figure 10: Implied time series



Notes: This figure plots the expected inflation, non-inflation disaster probability and inflation disaster probability implied by the historical values of price-earning ratio for the S&P 500 index, average nominal yields (average of one-, two-, three-, four-, and five-year nominal bond yields), and term spread (five-year yield minus one-year yield). The disaster probabilities are restricted to be non-negative. Data are available at monthly frequency from June 1952 to December 2011.

Table 1: Summary statistics of consumption disasters

Panel A: All countries	
Number of consumption disasters	89
Number of consumption disasters with high inflation	30
Percentage of consumption disasters with high inflation (%)	33.71
Panel B: OECD countries	
Number of consumption disasters	53
Number of consumption disasters with high inflation	17
Percentage of consumption disasters with high inflation (%)	32.08

Data from Barro and Ursua (2008). I exclude four OECD disasters and two non-OECD disasters in which inflation data are not available.

Table 2: Parameters

Panel A: Basic parameters	
Average growth in consumption (normal times) μ (%)	1.96
Average growth in dividend (normal times) μ_D (%)	3.63
Volatility of consumption growth (normal times) σ (%)	1.45
Leverage ϕ	3.5
Rate of time preference β	0.010
Relative risk aversion γ	3.0
Panel B: Inflation parameters	
Average inflation \bar{q} (%)	2.70
Volatility of expected inflation σ_q (%)	1.30
Volatility of realized inflation σ_p (%)	0.80
Mean reversion in expected inflation κ_q	0.09
Panel C: Non-inflation disaster parameters	
Average probability of non-inflation disaster $\bar{\lambda}_c$ (%)	1.83
Mean reversion in non-inflation disaster probability κ_{λ_c}	0.11
Volatility parameter for non-inflation disaster σ_{λ_c}	0.107
Minimum non-inflation disaster (%)	10
Power law parameter for non-inflation disaster	9
Panel D: Inflation disaster parameters	
Average probability of inflation disaster $\bar{\lambda}_{cq}$ (%)	1.03
Mean reversion in inflation disaster probability $\kappa_{\lambda_{cq}}$	0.11
Volatility parameter for inflation disaster $\sigma_{\lambda_{cq}}$	0.093
Minimum inflation disaster (%)	10
Power law parameter for inflation disaster	7

Table 3: Log consumption and dividend growth moments

Panel A: Consumption growth								
		No-Disaster Simulations			All Simulations			
	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
mean	1.91	1.64	1.95	2.26	-0.38	1.38	2.10	1.51
standard deviation	1.41	1.23	1.44	1.66	1.34	3.74	9.95	3.90
skewness	-0.48	-0.49	0.00	0.48	-6.72	-4.02	0.22	-8.51
kurtosis	3.49	2.20	2.80	3.94	2.48	21.73	49.73	107.47

Panel B: Dividend growth								
		No-Disaster Simulations			All Simulations			
	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
mean	1.78	2.42	3.50	4.59	-4.66	1.50	4.03	1.97
standard deviation	6.57	4.30	5.05	5.83	4.69	13.10	34.83	13.66
skewness	-0.01	-0.49	0.00	0.48	-6.72	-4.02	0.22	-8.51
kurtosis	5.26	2.20	2.80	3.94	2.48	21.73	49.73	107.47

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years and then aggregating monthly growth rates to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur.

Table 4: Inflation moments

	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
Mean	3.74	0.45	3.72	14.88	0.95	6.15	30.73	12.11
Standard deviation	3.03	1.78	2.94	16.79	1.91	5.73	33.65	23.52
AC(1)	0.66	0.61	0.84	0.93	0.65	0.87	0.94	0.94

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years and then aggregating monthly growth rates to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur. All numbers are in annual level terms.

Table 5: Nominal Yield Moments

Panel A: Average nominal bond yield								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
1-year	5.20	2.38	5.53	14.32	2.28	7.22	24.17	11.18
2-year	5.40	2.69	5.72	14.39	2.57	7.41	24.20	11.30
3-year	5.58	2.92	5.87	14.34	2.78	7.53	24.08	11.34
4-year	5.72	3.11	5.98	14.20	2.93	7.61	23.85	11.31
5-year	5.82	3.26	6.06	14.10	3.06	7.65	23.53	11.25

Panel B: Volatility of nominal bond yield								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
1-year	3.02	1.70	2.87	12.80	1.91	5.25	21.87	14.18
2-year	2.97	1.63	2.76	12.44	1.83	5.13	21.30	13.88
3-year	2.90	1.60	2.72	12.07	1.81	5.03	20.73	13.56
4-year	2.84	1.59	2.70	11.75	1.79	4.96	20.18	13.24
5-year	2.78	1.58	2.69	11.42	1.78	4.91	19.64	12.92

Notes: Panel A reports the average nominal bond yield and Panel B reports the volatility of the nominal bond yield. Data moments are calculated using monthly data from 1952 to 2011. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur. All yields are in annual terms.

Table 6: Bond Excess Return

Panel A: Average Bond Excess Return								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
2-year	0.47	-0.23	0.35	1.28	-0.56	0.33	1.78	0.47
3-year	0.91	-0.41	0.59	2.13	-0.92	0.56	2.91	0.77
4-year	1.14	-0.55	0.79	2.75	-1.19	0.75	3.75	1.00
5-year	1.37	-0.61	0.97	3.26	-1.36	0.90	4.41	1.21

Panel B: Standard Deviation of Bond Excess Return								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
2-year	1.80	1.29	1.76	5.62	1.39	2.91	9.81	5.01
3-year	3.41	2.49	3.36	9.42	2.68	5.36	16.27	8.55
4-year	4.74	3.66	4.99	12.24	3.96	7.62	21.19	11.44
5-year	5.97	4.80	6.66	14.64	5.25	9.76	25.46	14.08

Panel C: Sharpe Ratios								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
2-year	0.26	-0.07	0.19	0.56	-0.11	0.13	0.51	0.09
3-year	0.27	-0.07	0.17	0.50	-0.11	0.11	0.46	0.09
4-year	0.24	-0.07	0.15	0.45	-0.11	0.11	0.41	0.09
5-year	0.23	-0.07	0.14	0.41	-0.10	0.10	0.37	0.09

Notes: Data moments are calculated using annual data from 1952 to 2011. Short-term rates are constructed using one-year bond yield.

Table 7: Correlation between principal components and state variables

	PC1	PC2	PC3
expected inflation	0.96	0.14	0.22
non-inflation disaster risks	-0.29	0.13	0.91
inflation disaster risks	0.17	-0.95	0.13

Notes: This table reports the correlation between each principal component and each state variable in the model. I report the median value drawn from the subset of small-sample simulations having no consumption disasters.

Table 8: Campbell-Shiller long rate regression

Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
1-year	-0.57	-0.98	-0.30	2.47	-0.93	0.06	3.38	0.20
2-year	-0.74	-1.13	-0.38	2.61	-1.05	0.09	3.60	0.39
3-year	-1.14	-1.36	-0.47	2.73	-1.25	0.10	3.74	0.51
4-year	-1.44	-1.62	-0.57	2.81	-1.49	0.09	3.83	0.60
5-year	-1.68	-1.89	-0.66	2.88	-1.74	0.09	3.91	0.67

Notes: This table reports the coefficients of the Campbell-Shiller regression.

$$y_{t+h}^{\$, (n-h)} - y_t^{\$, (n)} = \text{constant} + \beta_n \frac{1}{n-h} \left(y_t^{\$, (n)} - y_t^{\$, (h)} \right) + \text{error},$$

where $h = 0.25$ and each row represents a bond with a different maturity (n). Data moments are calculated using quarterly data from June 1952 to December 2011.

Table 9: Cochrane-Piazzesi forward rate regression

Panel A: Coefficient								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
2-year	0.44	0.34	0.42	0.51	0.34	0.42	0.52	0.61
3-year	0.83	0.75	0.81	0.88	0.75	0.82	0.89	0.94
4-year	1.26	1.18	1.20	1.21	1.18	1.20	1.21	1.15
5-year	1.47	1.43	1.57	1.70	1.41	1.56	1.70	1.30

Panel B: R^2 -statistics								
Maturity	Data	No-Disaster Simulations			All Simulations			Population
		0.05	0.50	0.95	0.05	0.50	0.95	
2-year	0.16	0.02	0.17	0.55	0.02	0.12	0.47	0.02
3-year	0.17	0.03	0.18	0.53	0.02	0.12	0.45	0.01
4-year	0.20	0.03	0.18	0.49	0.02	0.13	0.42	0.01
5-year	0.18	0.03	0.18	0.44	0.02	0.13	0.39	0.01

Notes: This table reports the results from the second stage of the Cochrane-Piazzesi single factor regression. It reports the coefficient on the linear combination of forward rates on nominal bonds and the R^2 -statistics from regressing excess bond return on the single forward rate factor. I consider bonds with maturities of two, three, four and five years. Data are monthly from 1952 to 2011.

Table 10: Market moments

	No-Disaster Simulations				All Simulations			Population
	Data	0.05	0.50	0.95	0.05	0.50	0.95	
$E[R^{(0.25)}]$	1.25	0.59	1.80	2.40	-1.13	1.28	2.24	0.99
$\sigma(R^{(0.25)})$	2.75	0.95	1.38	2.45	1.05	1.82	3.67	2.45
$E[R^m - R^{(0.25)}]$	7.25	4.05	6.12	9.48	2.77	5.76	10.23	6.87
$\sigma(R^m)$	17.79	10.02	14.59	21.44	11.89	19.33	29.76	19.61
Sharpe ratio	0.41	0.31	0.42	0.56	0.14	0.31	0.49	0.35
$\exp(E[p - d])$	32.51	26.20	31.63	34.68	20.74	29.38	33.97	28.22
$\sigma(p - d)$	0.43	0.09	0.18	0.34	0.11	0.24	0.48	0.32
AR1($p - d$)	0.92	0.49	0.75	0.90	0.56	0.80	0.93	0.89

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. We also simulate 10,000 60-year samples and report the 5th-, 50th-, and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur. $R^{(0.25)}$ denotes the three-month Treasury Bill return where $R^{(0.25)} = R_t^{\$, (0.25)} \frac{P_{t+1}}{P_t}$. R^m denotes the return on the aggregate market, and $p - d$ denotes the log price-dividend ratio.

Table 11: Long-horizon regressions of returns on the price-dividend ratio (One-year holding period)

Panel A: Aggregate Market									
		No-Disaster Simulations			All Simulations				
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	-0.12	[-1.89]	-0.62	-0.35	-0.19	-0.51	-0.23	0.05	-0.17
R^2	0.07		0.10	0.20	0.31	0.00	0.08	0.25	0.07

Panel B: Bond Market									
		No-Disaster Simulations			All Simulations				
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.02	[1.19]	-0.15	-0.03	0.06	-0.14	-0.01	0.15	-0.01
R^2	0.02		0.00	0.03	0.38	0.00	0.02	0.22	0.00

Notes: This table reports the results from regressing one-year aggregate market excess returns and average nominal bond excess return on the price-dividend ratios. Data are annual from 1953 to 2010. For the data coefficients, I report *t*-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 12: Long-horizon regressions of returns on the linear combination of forward rates (One-year holding period)

Panel A: Aggregate Market									
			No-Disaster Simulations			All Simulations			
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.59	[3.39]	-2.98	0.76	3.05	-3.00	0.71	3.43	2.16
R^2	0.00		0.00	0.03	0.17	0.00	0.02	0.15	0.01

Panel B: Bond Market									
			No-Disaster Simulations			All Simulations			
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	1.15	[3.38]	0.92	1.15	1.34	0.78	1.14	1.45	1.26
R^2	0.19		0.06	0.24	0.57	0.01	0.13	0.45	0.01

Notes: This table reports the results from regressing one-year aggregate market excess returns and average nominal bond excess return on the linear combination of forward rates. Data are annual from 1953 to 2010. For the data coefficients, I report *t*-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 13: Long-horizon regressions of returns on the price-dividend ratio (Five-year holding period)

Panel A: Aggregate Market									
		No-Disaster Simulations			All Simulations				
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	-0.28	[-2.87]	-1.49	-1.08	-0.68	-1.49	-0.86	0.20	-0.68
R^2	0.13		0.21	0.51	0.72	0.00	0.27	0.66	0.24

Panel B: Bond Market									
		No-Disaster Simulations			All Simulations				
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.07	[1.83]	-0.51	-0.06	0.23	-0.53	-0.01	0.53	-0.04
R^2	0.09		0.00	0.07	0.50	0.00	0.06	0.42	0.00

Notes: This table reports the results from regressing five-year aggregate market excess returns and average nominal bond excess return on the price-dividend ratios. Data are annual from 1953 to 2010. For the data coefficients, I report *t*-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 14: Long-horizon regressions of returns on the linear combination of forward rates (Five-year holding period)

Panel A: Aggregate Market									
			No-Disaster Simulations			All Simulations			
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	2.83	[0.80]	-11.33	2.18	8.75	-11.70	2.20	11.61	8.56
R^2	0.02		0.00	0.09	0.48	0.00	0.07	0.43	0.04

Panel B: Bond Market									
			No-Disaster Simulations			All Simulations			
	Data	<i>t</i> -stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	2.09	[2.62]	1.23	3.16	4.92	0.91	3.49	6.28	4.94
R^2	0.12		0.04	0.32	0.70	0.01	0.25	0.65	0.04

Notes: This table reports the results from regressing five-year aggregate market excess returns and average nominal bond excess return on the linear combination of forward rates. Data are annual from 1953 to 2010. For the data coefficients, I report *t*-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.